Report Cover Page

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Optimal allocation of resources to emergency response actions for invasive species
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Summary
This project aims to develop new methods to assess and guide the development of monitoring and surveillance systems for invasive pests, pathogens and diseases. The project will use Red Imported Fire Ants and fruit fly surveillance systems as case studies. The researchers are working in close collaboration with state agencies to develop methods that will improve the efficiency and effectiveness of their efforts. This project documents the outputs from the first year's work by one of the post-doctoral researchers (Cindy Hauser), and the outlines of proposed work by another post-doc (Peter Baxter) and PhD student (Tracy Rout). They will work jointly to develop and test methods for efficient surveillance. The report also include appendices that outline the development of mathematical tools to support the methods.

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Optimal allocation of resources to emergency response actions for invasive species; 06/04

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1. **Executive Summary**

With weeds, pests and diseases causing billions of dollars worth of damage to the Australian landscape, it is vital that resources spent to manage these pests are expended efficiently. This project aims to provide decision support tools to managers of monitoring and control programs, to assist in the allocation of resources amongst activities.

We model the processes of detection, control, containment and/or eradication of a single pest species and use the economic outcomes of each possible management scenario to find the most efficient strategies. Research questions include:

- How much monitoring effort should be applied to detect a pest before it outweighs the economic impact of an undetected pest?
- How should limited surveillance resources be targeted across a landscape when the probability of pest presence varies according to environmental conditions? What if the accuracy of the surveillance method also varies across environments?
- For how long should absence data be collected before a pest can be declared eradicated with confidence?

We have already found mathematical expressions of the optimal strategies that correspond to the above questions. These strategies will be demonstrated in case studies of red imported fire ants, fruit flies, hawkweed and bitterweed.

Future research will adapt these strategies to account for limited data and ecological knowledge, ensuring that management is robust to a range of plausible scenarios.

2. Introduction

Incursions of weeds, pest and diseases cause great economic and environmental damage to the Australian landscape. There is a clear need for rational and rapid response procedures for incursions, and for decisions to be made in the face of uncertainty. In this project, we develop tools that support decision-making for the monitoring and control of pest species. We use a decision-theoretic framework to evaluate the costs and likely outcomes of various management regimes. This allows us to determine where in space and time a manager's resources should be targeted to most effectively detect, contain and/or eradicate a pest with confidence.

This research has been split into three sub-projects, each with a full-time post-doctoral researcher or PhD student attached. The first sub-project focuses on the successful detection and control of a pest species over space. The optimal monitoring effort to be expended in any area is a trade-off between the cost of monitoring and the savings made by detecting the pest early. However, when limited monitoring resources are to be distributed across space, the relative likelihood of pest presence in each different environment is used to target monitoring and minimise the probability of failing to detect the pest in any area.

Declaring eradication is the focus of the second sub-project, with a case study on the eradication of bitterweed in Queensland. Data on pest absence must be collected over time for eradication to be declared with confidence, but this surveillance incurs a cost. Here the crucial economic trade-off is between the cost of surveillance and the cost of the pest spreading after eradication is prematurely declared. An initial manuscript will be expanded to consider issues of spatial arrangement and robustness.

The third sub-project will offer direct support for the detection, management and eradication of red imported fire ants in south-east Queensland. Here there is a focus on the last rare colonies after successful widespread control. How should surveillance resources be directed to ensure that the pest is completely eradicated? Will new surveillance technologies and probability maps make improvements that reach beyond the cost of their development?

The following three chapters of this document report on the progress of each sub-project separately but the common themes are clear. We model the monitoring and control of pest species and use the economic outcomes of each plausible scenario to support decision-making and allocation of resources. These decision-making tools will not be specific to the case studies used, and will instead have utility for a wide range of pest management problems. Further links and collaborations amongst the sub-projects will form as the research progresses.

3. Project 1: Allocation of monitoring resources for pest management over space

Cindy Hauser and Michael McCarthy

In this project we focus on the targeted management of a pest or disease (hereon referred to as a 'pest'). Monitoring over space to detect and control the pest could be distributed uniformly or concentrated in specific areas. Monitoring might be concentrated if there is reason to believe that the pest is more likely to be present in a particular area, due to the habitat type, previous incursions or other factors. However, there will still be some (albeit smaller) probability that the pest occurs at other locations in the landscape. How should monitoring effort be distributed amongst different patches of space, given the differential probabilities of pest presence in those patches? (Patches could represent different types of vegetation, crops or catchment areas.) We have answered this question by examining the economic consequences of the various possible allocations of monitoring effort.

We model a system of *n* patches, where each patch has a specified probability of pest presence. We leave it to the manager to decide how to delineate patches in the landscape, such that the probability of pest presence and effectiveness of monitoring can be reasonably assumed to be constant within each patch. The effectiveness of investing one unit of monitoring effort in a patch can vary between patches as a function of the area of the patch, the ease of pest detection given the terrain, and other factors. If an existing pest is successfully detected by the monitoring program then it is controlled, incurring a cost c_D. If the monitoring program fails to detect an existing pest then a much larger cost c_U is incurred as the pest spreads, causing more damage and becoming more difficult to eradicate. These costs incurred are the same at each patch, since we assume that incursions occur on a smaller spatial scale than the scale at which patches are defined. As more monitoring effort is invested in a patch, the probability that an existing pest will persist undetected declines (Figure 3.1).

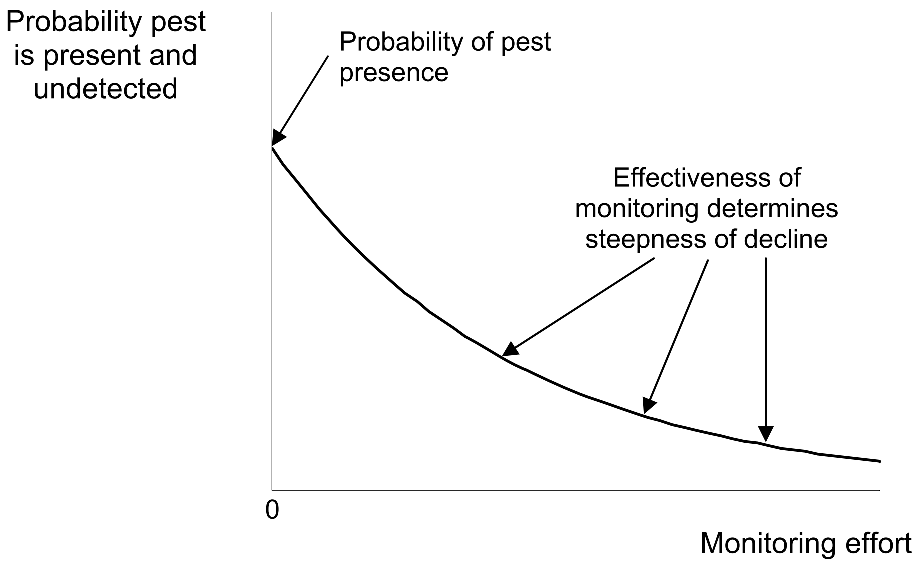

Figure 3.1. The probability that a pest is present and detected as a function of monitoring effort.

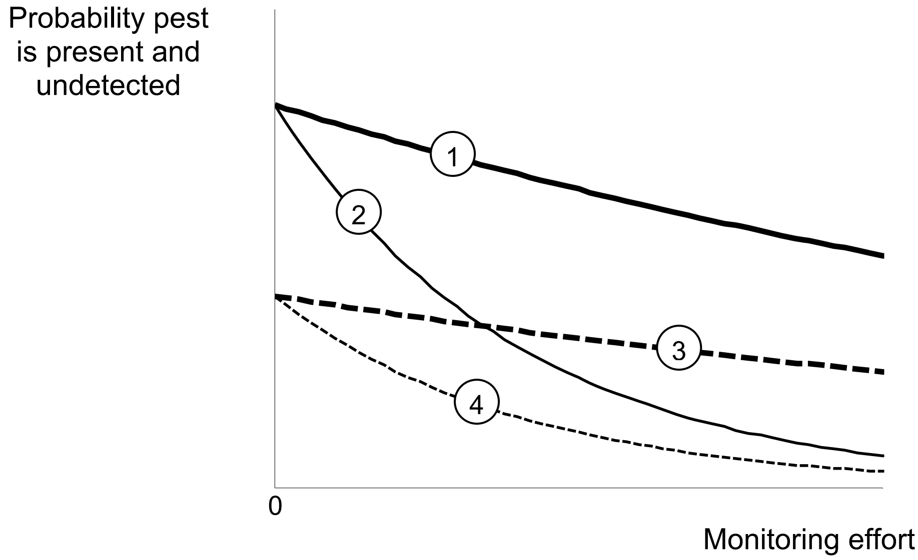

Figure 3.2 The probability that a pest is present and undetected, as a function of monitoring effort, for four example patches.

In different patches, the differing probabilities of pest presence and effectiveness of monitoring interact to produce differing probabilities of failing to detect an existing pest as a function of monitoring effort expended.

In Figure 3.2, the patches have:

1. a high probability of pest presence and low effectiveness of monitoring;
2. a high probability of pest presence and high effectiveness of monitoring;
3. a low probability of pest presence and low effectiveness of monitoring;
4. a low probability of pest presence and high effectiveness of monitoring.

All equations used to derive the results for this project (1) are included in Appendix 6.1.

3.1. Minimising expected costs of monitoring and control

We first aim to minimise the total expected costs of monitoring and control, where:

monitoring costs = number of monitoring units × cost per monitoring unit

and for each patch,

expected control cost = probability pest is present × cost of control,

cost of control = probability pest is detected × c_D + probability pest is not detected × c_U,

where c_D and c_U have been defined as the costs incurred for damage and pest control when the pest is and is not detected, respectively.

The optimal monitoring allocation to each patch *i* that minimises the expected costs of monitoring and management is

$$m_i^* = \begin{cases} \dfrac{1}{\lambda_i}\ln\left[\dfrac{(c_U - c_D)p_i\lambda_i}{c_M}\right], & p_i\lambda_i \geq \dfrac{c_M}{c_U - c_D} \\ 0, & p_i\lambda_i < \dfrac{c_M}{c_U - c_D} \end{cases}$$

where

λ_i is the "effectiveness" of monitoring in patch *i,*
p_i is the probability that the pest is present in patch *i*,
c_M is the unit cost of monitoring.

The monitoring effectiveness λ_i is the mean Poisson rate of pest detection using a single monitoring unit per unit area (such that the probability of failing to detect an existing pest is $\exp(-\lambda_i)$). Monitoring can encompass active, targeted surveillance methods that can be altered in intensity and potentially also passive methods, with the use of public awareness programs allowing some control over monitoring intensity. (ACERA project 06/05 will model passive and 'syndromic' surveillance more explicitly.) In the passive case, the effectiveness parameter λ_i might take into account proximity to towns, roads and other measures of human access.

The probability that the pest is present, p_i, can incorporate any information that is considered useful, such as historical evidence of pest presence or absence, vegetation mapping and pest dispersal patterns. We leave it to practitioners to decide, in any given pest management scenario, what the relevant factors are in estimating the probability of pest presence.

It may be optimal to allocate zero monitoring effort to a patch, subject to the condition on the right of the equation. This may occur because the probability of pest presence is low (p_i small), the monitoring method is ineffective (λ_i is small) or the cost of monitoring is large relative to savings that early pest detection brings to control ($c_M/(c_U - c_D)$ is large). In these cases the likelihood and benefits of pest detection are overwhelmed by the costs of monitoring.

In other cases, there is a positive optimal allocation of monitoring resources to the patch. The optimal monitoring effort increases with $c_U - c_D$, the improvement in control costs made by detecting the pest early, and decreases as the unit cost of monitoring c_M increases. The higher the probability that the pest is present (p_i), the larger the optimal monitoring effort for the patch. The influence of the monitoring effectiveness λ_i, on the optimal allocation of monitoring effort to patch *i* is more complicated. When monitoring is very ineffective, there is no benefit to monitoring and the optimal allocation is zero. The optimal allocation increases to a maximum value over intermediate values of effectiveness, then declines asymptotically to zero as effectiveness continues to increase. That is, for highly effective monitoring, only a nominal amount of effort is required to successfully detect an existing pest.

3.2. Minimising expected cost of control subject to a monitoring budget

If the resources available for monitoring are less than those required for the optimal solution in §3.1, then we are faced with distributing monitoring resources amongst the patches. In this case we seek to minimise the expected cost of control subject to a monitoring budget. A rearrangement of this objective function shows that this is equivalent to minimising the sum over all patches of the probability that the pest is present and undetected in each patch (as displayed in Figure 3.2). This is a reasonable result, since the most costly scenario in control is that the pest is present but undetected. Thus, we seek to allocate monitoring effort to minimise the incidence of this scenario.

The optimal allocation of monitoring effort lies in the gradient of the curve for each patch in Figure 3.2. The steeper the gradient, the more effective monitoring is at reducing the probability of leaving a pest undetected. As monitoring is allocated to a patch, the gradient of the line becomes less steep and the value of adding an extra unit of monitoring diminishes. There may now be a second patch which offers equivalent improvements for each unit of monitoring invested, and the budget will then be allocated to the two patches. This procedure continues, with further patches receiving monitoring funds, until the budget is exhausted. Thus, the optimal monitoring allocation to a budget does *not* depend on the cost of control: it depends only on the total monitoring budget, the probability of pest presence in each patch, and the effectiveness of monitoring in each patch.

Example 3.2.1. Equal detection and area but differing probability of pest presence

Assume we have two patches, each of the same area. Investment in monitoring is equally effective in each patch, with $\lambda_1 = \lambda_2 = 0.01$. However there is a 1% probability of pest presence in the first patch and 3% probability of pest presence in the second patch. We have a budget of 200 monitoring units to distribute between the two patches.

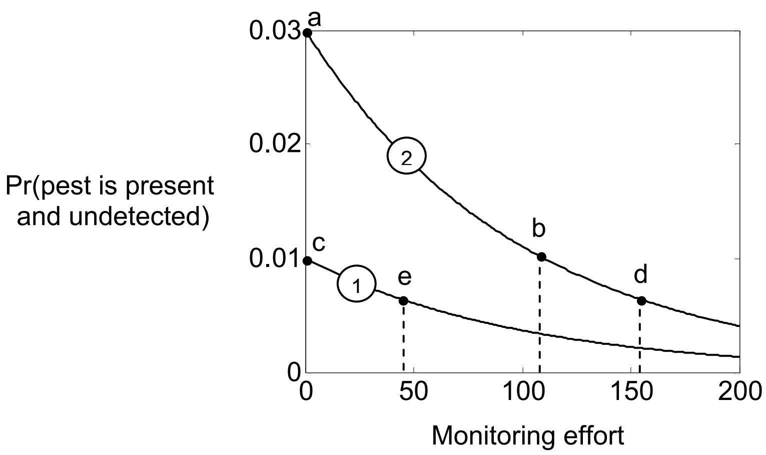

Figure 3.3. Probability that the pest is present and undetected as a function of monitoring effort for the two patches in Example 3.2.1

When we begin with zero monitoring effort in each patch, the probability curve for patch 2 is steeper and so we make it the first priority (point a, Figure 3.3). It remains the most cost-effective patch until 110 monitoring units have been allocated, and the gradient of the curve here is equal to that for patch 1 (b, c). The remainder of the monitoring budget is allocated to both patches, in accordance with their gradients, until the budget of 200 units is exhausted.

The final optimal allocation is 45 units to patch 1 and 155 units to patch 2 (d, e). Patch 1 is allocated greater monitoring resources because the pest is more likely to exist there, but some effort should still be expended on patch 1 to determine its status. Note that if it is optimal to allocate monitoring units to both patches, the difference in the number of monitoring units between the two patches remains 110 (in this example), regardless of the budget.

Example 3.2.2. Differing areas and probabilities of pest presence

In this example we consider four patches, with different combinations of area and probability of pest presence (Figure 3.4). Patches 1 and 4 have an area of 1 unit, while patches 2 and 3 have an area of 2. Patches 1 and 2 have a high probability of pest presence, 0.04, compared to patches 3 and 4 which have probability 0.005 of pest presence. The effectiveness of monitoring per unit area is constant over all patches at 0.01.

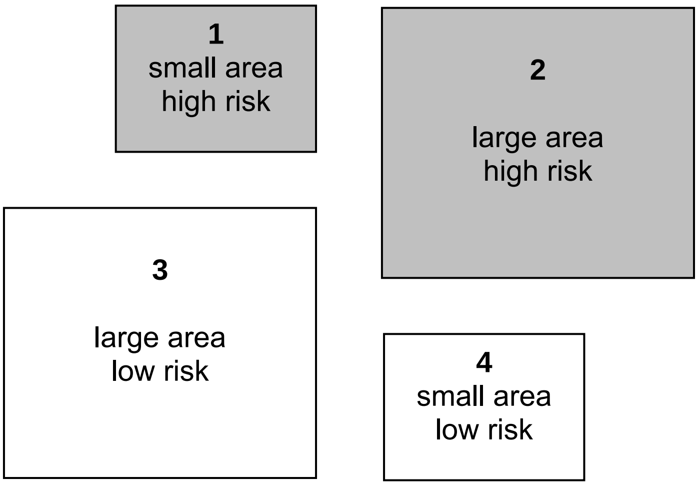

Figure 3.4. Diagram of patches for Example 3.2.2, with relative areas and risks of pest presence labelled.

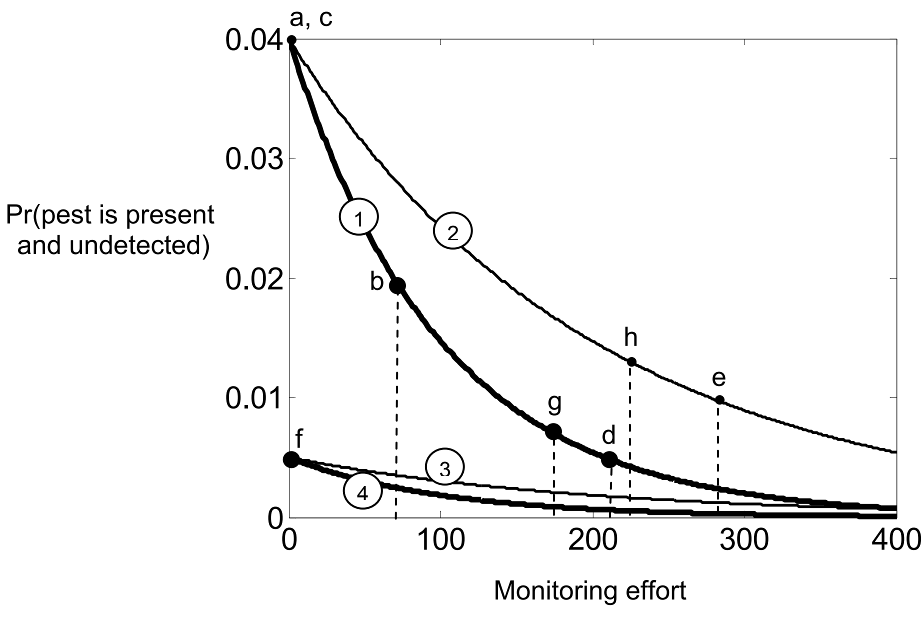

Figure 3.5. Probability that the pest is present and undetected as a function of monitoring effort for each of the four patches in Example 3.2.2.

Then the monitoring effectiveness in each patch is:

$$\lambda_1 = \frac{0.01}{1} = 0.01,\ \lambda_2 = \frac{0.01}{2} = 0.005,\ \lambda_3 = 0.005,\ \lambda_4 = 0.01.$$

Patch 1 is the most cost effective (point a, Figure 3.5), so monitoring effort is first allocated to it. When 69 monitoring units have been allocated to patch 1, patch 2 is equally cost-effective (b, c), and further monitoring effort is allocated to both of these patches. The third most cost-effective patch, patch 4, matches these two for cost effectiveness when 208 units have been allocated to patch 1 and 277 units have been allocated to patch 2 (d, e, f). This would exceed the budget of 400 monitoring units and so no monitoring effort is allocated to patches 3 and 4. Instead the budget is allocated only to patches 1 and 2, with 180 and 220 units respectively (g, h). Note that even though patch 1 was prioritised first, patch 2 was ultimately subject to more monitoring effort. This is because patch 2 had a greater area, necessitating more monitoring units to achieve the same reduction in undetected pests.

Instead we might allocate the budget uniformly over the each unit area. Then patches 1 to 4 would be allocated 67, 133, 133 and 67 monitoring units each, respectively. If we know the costs of control, c_U and c_D, then we can calculate the difference in expected total costs between monitoring uniformly over space and using a habitat map or other prior information to target monitoring optimally over space. If in this example c_U = 100000 and c_D = 1000, then a saving of about 1600 units can be made in pest control by concentrating monitoring effort in patches 1 and 2 as above.

3.3. Further extensions

Value of habitat mapping

In Example 3.2.2, we determined the expected saving to be made by targeting monitoring as a function of the probability of pest presence and effectiveness of monitoring over space. When little is known about the differential risks across space, a similar framework could be used to determine the expected improvement that building a habitat map would provide, over simply monitoring uniformly over space. Then a manager could decide whether to invest in the construction of such a habitat model, or to monitor for the pest without any extra knowledge of its likely location.

Spatial autocorrelation

In the current framework, pest presence or absence in each patch is independent of presence or absence in any other patch. Correlation in patch status (due to spatial configuration or other factors) could alter the optimal allocation of monitoring effort. For example, allocation of high monitoring intensity to two highly correlated patches may be a suboptimal use of resources.

Robustness of the optimal monitoring allocation

It is unlikely that the probability of pest presence and effectiveness of monitoring will be known for each patch in space. If a patch is incorrectly classified as having a very low probability of pest presence then zero monitoring effort might be allocated to the patch (as in Example 3.2.2). If the actual probability of pest presence is medium or high, then there is a substantial risk that the pest will be present and undetected in that patch. A more robust solution would be to allocate monitoring effort uniformly over space, but setting the problem in a formal decision theory framework is necessary to find an allocation of monitoring effort that will yield acceptably low costs over a range of plausible scenarios.

Application to case studies
We will apply these methods to one or more case studies in plant biosecurity to demonstrate their potential use for planning pest management and monitoring. Possible case studies include the monitoring and management of fruit flies in the Fruit Fly Exclusion Zone in northern Victoria and of hawkweed in the alpine region of Victoria. Results from the case studies may indicate the relative importance of robust decision-making, spatial autocorrelation and the value of habitat mapping and direct further research.

3.4. Summary

This project sets the monitoring and control of a single pest species in an economic framework. We determine the optimal allocation of monitoring resources over space to detect new incursions and eradicate them. This optimal allocation depends on the probability of pest presence and the effectiveness per unit monitoring effort in each patch.

When monitoring resources are unlimited, there is still an optimal finite level of monitoring effort which trades the economic consequences of failing to detect a pest against the cost of monitoring. If the monitoring method is ineffective and/or expensive, then its cost may overwhelm the potential benefits and zero monitoring in the patch is optimal. As the probability of pest presence increases, so does the optimal expenditure on monitoring.

When monitoring resources are restricted to a budget, the optimal resource allocation depends crucially on the probability of pest presence and effectiveness of monitoring in each patch, not on the costs of controlling the pest. Patches that offer the greatest reduction per unit monitoring effort in the probability of failing to detect an existing pest are prioritised.

4. Project 2: Optimal robust monitoring of invasive species

Tracy Rout and Michael McCarthy

4.1. When should we declare eradication of an invasive species?

The question of when to stop monitoring and declare eradication of an invasive species is usually answered by using arbitrary confidence thresholds or ideas about seed bank longevity (Regan et al. 2006). Regan et al. (2006) take an economic approach to this question, framing it in a decision theoretic way. They find the stopping time (based on the number of previous consecutive absent surveys) that will minimise the net economic cost, as defined by:

$$NEC_n = (n - 1)C_s + C_e[p(1 - q)]^n ,$$

where NEC_n is the net economic cost of stopping after n consecutive absent surveys, C_s is the cost of each survey, C_e is the expected cost if eradication is declared prematurely and the species expands its range, p is the annual probability that the species will remain present, and q is the probability of detecting the species given that it is present. They minimise this equation to find the optimal stopping time, i.e. the optimal number of consecutive absent surveys n^* after which monitoring should stop and eradication should be declared. They then use stochastic dynamic programming to take into account the possibility that the weed may be detected in future surveys, incurring further costs of surveying and possible escape and damage.

Although this work represents a new way of thinking about how we approach setting guidelines for invasive species eradication, its practicality is reduced by the data requirements of the model. The parameters p (the annual probability of persistence) and q (detectability), are difficult to estimate for many invasive species. We can eliminate the need to estimate these parameters by instead using the presence-absence sighting record of the species. This is done by incorporating the equation from Solow (1993a) into the decision-making framework of Regan et al. (2006), which is described in detail in Appendix 8.2.

We first examine the analytical solution of the new equation, and attempt to find a rule of thumb for when to declare eradication. We then use stochastic dynamic programming to find an exact optimal solution that incorporates the possibility that the weed may be seen in future surveys. We apply these methods to the example of *Helenium amarum*, which enables a direct comparison with the results in Regan et al. (2006). Some preliminary findings are described in Appendix 6.2.

This basic framework can be modified and extended in a number of ways. It could be useful to modify Solow's equation for use in declining populations where the pre-extinction sighting rate declines, as in Solow (1993b). As the *Helenium amarum* sighting data has a variable number of surveys per year, it would be useful to modify Solow's equation for use with frequency data, as in Burgman et al. (1995). Another extension would be to apply the entire framework to the problem of monitoring a species of conservation concern.

4.2. Declaring eradication for multiple infestations

The first part of this project examines the eradication of an invasive species at a single site, given data on the presence or absence of the species at that site. The second part will expand upon this to look at the eradication of an invasive species from a number of sites. Instead of a presence/absence sighting record, we will instead work with a record of the number of occupied sites over time. For a given monitoring strategy—where a certain proportion of the sites are surveyed in each time step—we can find the optimal time to stop monitoring and declare eradication. Expanding this problem across multiple sites will make it more general, and applicable to a wider range of case studies.

4.3. Robust decision-making

The methods mentioned so far have focused on finding optimal management decisions given a specified model of the system. These optimal decisions are not universal—if we alter the system model the optimal decision will most likely be different. If the model we use does not sufficiently approximate reality, the prescribed decision will not have the outcome we intend, and may even be detrimental. Instead of finding management decisions that are optimal for a specific set of circumstances, it may be more practical to find robust decisions that perform well under a large range of circumstances. To examine this perspective, the project will also examine robust solutions to the previous two problems. This may lead to results that are analogous to an earlier contribution to this ACERA project (Thompson and McCarthy, in prep., see previous progress report and Appendix 6.3 in this document).

4.4. References

Burgman, M. A., R. C. Grimson, and S. Ferson. 1995. Inferring Threat from Scientific Collections. *Conservation Biology* **9**:923-928.

Regan, T. J., M. A. McCarthy, P. W. J. Baxter, F. D. Panetta, and H. P. Possingham. 2006. Optimal eradication: when to stop looking for an invasive plant. *Ecology Letters* **9**:759-766.

Solow, A. R. 1993a. Inferring Extinction from Sighting Data. *Ecology* **74**:962-964.

Solow, A. R. 1993b. Inferring extinction in a declining population. *Journal of Mathematical Biology* **32**:79-82.

5. Project 3: Optimal choice and timing of monitoring strategies

Peter Baxter and Hugh Possingham

5.1. Introduction

When considering the ongoing surveillance of an invasion, we can expect the following relationships:
1) The more resources (effort or money) we invest in surveillance, the more likely we are to detect an incidence of invasion, and the sooner we should detect it;
2) The sooner we detect an incidence, the less future costs we face (we catch it earlier, therefore it is easier to eradicate and less likely to have spread);
3) The more effort we expend (in time or intensity) without detecting an occurrence, the more likely that eradication has occurred.

There are thus multiple possible trade-offs involving surveillance method, effort and timing. If we set a deadline (i.e., a management "time horizon") we can seek general guidelines to make appropriate tradeoffs which will meet management objectives within that timeframe. These objectives could be for example:
- maximise our confidence in having achieved eradication;
- minimise the probability of missing an occurrence; or
- minimise the overall expected costs (sum of expected survey and escape costs).

We will tackle these questions in a decision theory framework by defining curves that describe the above relationships (1-3); and using these curves to find optimal strategies analytically. We will also consider uncertainty in the exact form of each curve and how the nature and extent of uncertainty might change our recommendations. We will then examine the applicability of the approach using a case study.

5.2. Case study: red imported fire ants

Red imported fire ants *Solenopsis invicta* were first recorded in south-east Queensland in February 2001. Native to South America, fire ants are capable of widespread establishment once introduced elsewhere (Calcott and Collins 1996), and have had serious environmental and societal impacts, notably in the southern USA. The potential reduction in Australian biodiversity due to *S. invicta* is listed as a Key Threatening Process under the 1999 Commonwealth Environment Protection and Biodiversity Conservation Act. Their detection in Australia therefore prompted the establishment of an eradication program. As this eradication program progresses and colonies become rarer, it is likely that the ability to detect colonies will also decline. Therefore there is a need to inform managers' choices regarding future surveillance.

In Queensland, there are two main fire-ant surveillance strategies: active (targeted surveys of likely areas of occurrence) and passive (relying on public reporting). These can in turn be further sub-categorised, with each sub-category having its own costs and efficiencies. For example, active surveillance could consist of searching by foot, all-terrain vehicle or helicopter; whereas passive surveillance could reflect different intensities of communication and advertising. Management decisions regarding how best to allocate surveillance resources can be addressed effectively using the above decision-theory framework. The approach can be extended further, to incorporate the possible emergence of better surveillance techniques in the future, such as sniffer-dogs or unmanned aerial vehicles; or to allow further resolution of the relationships (1-3) above, reflecting different habitats (probability of occurrence), land use and human population densities (probability of detection).

Two computer models produced for the eradication program assisted in understanding and addressing the invasion process. Scanlan and Vanderwoude (2004) developed a model of spatial spread which examines patterns of local and long-distance fire-ant invasion across a regular grid of cells ("cellular automata"). A predictive habitat model (Fire Ant Habitat Information System; Robert George, unpublished report) has facilitated more targeted surveillance, for example by selectively targeting 50% of an area, 98% of the occurrences within the total area could be detected. In order to assess more quantitatively whether the eradication program is indeed on track, the Fire Ant Control Centre is now considering revising and combining the two previous modelling approaches in a single model, which would incorporate spread dynamics with occurrence probability and also have the capacity to integrate new data to revise predictions using a Bayesian framework.

The production of such a model requires considerable effort, so again there arises a trade-off between the resources required in its development and the resultant improvement in predictability. This trade-off has obvious similarities with investing in development of new methods to increase detection success as discussed above. The time-lag involved in their development may or may not be worth the diversion of management funds from other activities – this will depend on their future success (relative to the performance of current methods), which is itself uncertain. Again, the decision-theoretical framework proposed above will address how to allocate management resources optimally among these alternative approaches, in order to attain eradication with maximum certainty and efficiency; and furthermore how these optimal decisions may change as eradication is approached.

5.3. References

Callcott AMA & HL Collins. 1996. Invasion and range expansion of imported fire ants (Hymenoptera: Formicidae) in North America from 1918–1995. Florida Entomologist 79:240–251.

Scanlan JC & C Vanderwoude. 2006. Modelling the potential spread of Solenopsis invicta Buren (Hymenoptera: Formicidae) (red imported fire ant) in Australia. Australian Journal of Entomology 45:1–9.

6. Appendix

6.1. Equations used in §3.1-3.2

We define

n	the total number of patches in the landscape.
p_i	the probability that the pest is present somewhere in patch i. This could be an increasing function of the area of the patch.
λ_i	the "effectiveness" of monitoring in patch i. This may be a decreasing function of the area of the patch. $e^{-\lambda}$ is the probability of failing to detect an existing pest using one unit of monitoring effort.
m_i	the monitoring effort invested in patch i.
M	the total monitoring budget
c_M	the cost of one unit of monitoring effort
c_D	the cost of pest management if the pest is detected and controlled.
c_U	the cost of pest management if the pest is not detected and consequently spreads to a larger area, $c_U > c_D$.

3.1 Minimising expected costs of monitoring and control

The expected cost of controlling the pest over n patches is the cost of control in each patch, summed over all patches i:

$$T_1(\mathbf{m}) = \sum_{i=1}^{n} p_i \left[c_D \left(1 - e^{-\lambda_i m_i} \right) + c_U e^{-\lambda_i m_i} \right].$$

The expected cost of control in each patch i is the probability of pest presence multiplied by the cost of pest management. The cost of pest management is c_D if the pest is detected (with probability $1 - e^{-\lambda_i m_i}$) and c_U if monitoring efforts fail to detect the pest (with probability $e^{-\lambda_i m_i}$). We can rearrange this cost function to become

(1) $$T_1(\mathbf{m}) = c_D \sum_{i=1}^{n} p_i + \left(c_U - c_D \right) \sum_{i=1}^{n} p_i e^{-\lambda_i m_i}.$$

The first term indicates the minimum expected cost of control that must be incurred if all pests are detected and controlled, while the second term gives the additional costs incurred in patches where the pest is present but not detected.

The cost of monitoring over all patches is $\sum_{i=1}^{n} c_M m_i$, so the total cost of monitoring and control is:

$$T_2(\mathbf{m}) = c_M \sum_{i=1}^{n} m_i + c_D \sum_{i=1}^{n} p_i + \left(c_U - c_D \right) \sum_{i=1}^{n} p_i e^{-\lambda_i m_i}.$$

The minimum expected cost can be determined by setting the partial derivative of the cost function (with respect to each allocation m_i) to zero:

$$\left. \frac{\partial T_2}{\partial m_i} \right|_{\mathbf{m}=\mathbf{m}^*} = c_M - (c_U - c_D) p_i \lambda_i \exp(-\lambda_i m_i^*) = 0.$$

This equation can be rearranged to give the optimal allocation of monitoring effort m_i in each patch:

(2) $$m_i^* = \frac{1}{\lambda_i}\ln\left[\frac{(c_U - c_D)p_i\lambda_i}{c_M}\right].$$

This solution is only meaningful if $m_i^* \geq 0$, i.e.

(3) $$p_i\lambda_i \geq \frac{c_M}{c_U - c_D}.$$

If this condition does not hold, then the costs of monitoring overwhelm the expected benefits of early detection and control, so that the optimal monitoring allocation to patch *i* is $m_i^* = 0$. The total expected cost of monitoring and control in a single patch *i* is demonstrated in Figure 6.1 for two scenarios: in the first, condition (3) is violated and the monitoring effort that minimises costs is $m_i^* = 0$; in the second scenario condition (3) holds and there exists an intermediate monitoring effort m_i^* that minimises total expected costs (given in equation 2).

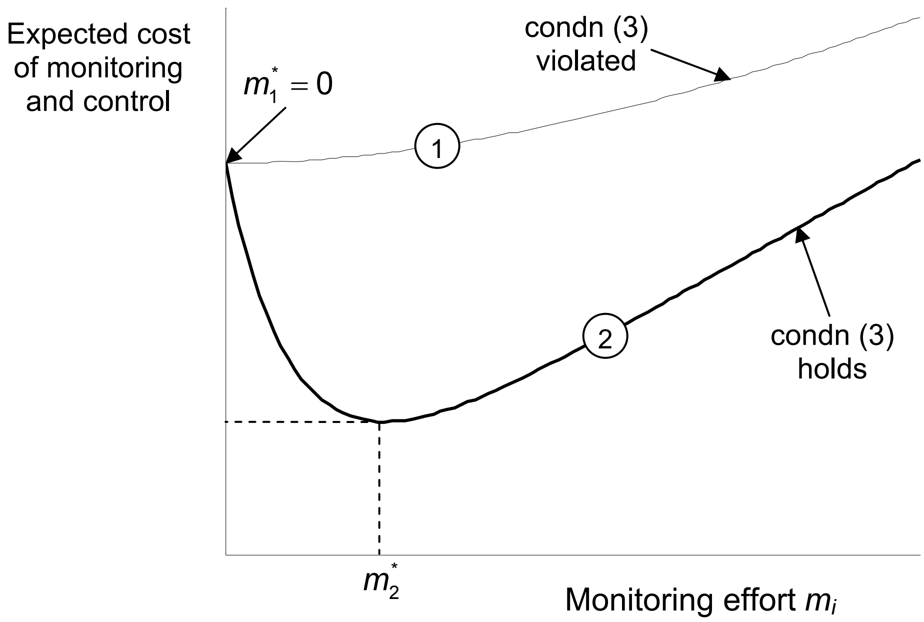

Figure 6.1. The expected costs of monitoring and control as a function of monitoring effort for two example patches.

3.2 Minimising expected cost of control subject to a monitoring budget

In this case we seek to minimise the expected cost of control, $T_1(\mathbf{m})$, subject to a monitoring budget M, such that

$$\sum_{i=1}^{n} c_M m_i \le M \qquad \text{or} \qquad \sum_{i=1}^{n} m_i \le \frac{M}{c_M} = M'.$$

Given that our decision variables are the monitoring allocations m_i, i = 1, 2, …, n, an equivalent problem to minimising $T_1(\mathbf{m})$ in eqn 1 is to minimise

$$T_3(\mathbf{m}) = \sum_{i=1}^{n} p_i \exp\left(-\lambda_i m_i\right)$$

The ith term is the probability that the pest is present but undetected in patch i, given that monitoring effort m_i was invested in this patch. However the sum over all i can not be directly interpreted as a probability, and may be greater than one. We set

Pr(undetected pest in patch i) = $f_i(m_i) = p_i \exp(-\lambda_i m_i)$.

As monitoring effort in a patch increases, the probability that an existing pest will be left undetected decreases as a function of monitoring effectiveness λ.(Figures 3.1 & 3.2). The gradient of this function indicates how efficiently we can reduce the probability of failing to detect a pest for each unit of effort expended:

$$\frac{\partial f_i}{\partial m_i} = -\lambda_i p_i \exp\left(-\lambda_i m_i\right).$$

Note that this cost-effectiveness of monitoring is a function of both the effectiveness of monitoring λ and the probability of pest presence p. The optimal allocation of monitoring effort is found by prioritising patches with the steepest gradient until the budget is spent.

Example 3.2.1
We have two patches, each of the same area. Investment in monitoring is equally effective in each patch, with $\lambda_1 = \lambda_2 = 0.01$. However the probability of pest presence differs in each patch, with $p_1 = 0.01$ and $p_2 = 0.03$. The budget is $M' = 200$. Then

$$\frac{\partial f_1}{\partial m_1} = -\lambda_1 p_1 \exp\left(-\lambda_1 m_1\right) = -0.0001 \exp(-0.01 m_1),$$
$$\frac{\partial f_2}{\partial m_2} = -0.0003 \exp(-0.01 m_2).$$

Initially, allocation to patch 2 is most effective, since

$$\left.\frac{\partial f_2}{\partial m_2}\right|_{m_2=0} = -0.0003 < -0.0001 = \left.\frac{\partial f_1}{\partial m_1}\right|_{m_1=0}.$$

Funds should be allocated to patch 2 until cost-effectiveness diminishes to the level of patch 1:

$$\frac{\partial f_2}{\partial m_2} = -0.0003\exp(-0.01m_2) = -0.0001 = \left.\frac{\partial f_1}{\partial m_1}\right|_{m_1=0}$$
$$\Rightarrow m_2 = 100\ln 3 \approx 110.$$

Therefore the first 110 units of monitoring effort should be allocated to patch 1. The remaining 90 units are allocated between patches 1 and 2 so that the cost-effectiveness of monitoring at each patch is equal:

$$\frac{\partial f_1}{\partial m_1} = \frac{\partial f_2}{\partial (m_2 + 110)}$$ subject to $m_1 + m_2 = 90$.

The solution to this set of equations is $m_1 = m_2 = 100 - 50 \ln 3 \approx 45$. Thus, the optimal allocation of effort between the patches is $m_1^* = 45$, $m_2^* = 110 + 45 = 155$.

Example 3.2.2
We have four patches, with different combinations of area and probability of pest presence. Patches 1 and 4 have area 1, while patches 2 and 3 have area 2. Patches 1 and 2 have a high probability of pest presence, 0.04, compared to patches 3 and 4 which have probability 0.005 of pest presence. The effectiveness of monitoring per unit area is constant over all patches at 0.01. Then the monitoring effectiveness in each patch is:

$$\lambda_1 = \frac{0.01}{1} = 0.01,\ \lambda_2 = \frac{0.01}{2} = 0.005,\ \lambda_3 = 0.005,\ \lambda_4 = 0.01.$$

The cost effectiveness of monitoring in each patch is

$$\frac{\partial f_i}{\partial m_i} = \begin{cases} -0.0004e^{-0.01m_1}, & i = 1 \\ -0.0002e^{-0.005m_2}, & i = 2 \\ -0.000025e^{-0.005m_3}, & i = 3 \\ -0.00005e^{-0.01m_4}, & i = 4 \end{cases}$$

It is most cost effective to first allocate resources to patch 1. Patch 2 is the second most cost-effective patch to target, and it matches the first patch when

$$\frac{\partial f_1}{\partial m_1} = -0.0004e^{-0.01m_1} = -0.0002 = \left.\frac{\partial f_2}{\partial m_2}\right|_{m_2=0}$$
$$\Rightarrow m_1 = 100\ln 2 \approx 69.$$

If we have a budget of 400 monitoring units, then we should allocate the first 69 units to patch 1, then allocate further units to patches 1 and 2 until patch 4 becomes equally cost effective:

$$\frac{\partial f_1}{\partial (m_1+69)} = \frac{\partial f_2}{\partial m_2} = -0.00005 = \left.\frac{\partial f_4}{\partial m_4}\right|_{m_4=0}$$
$$\Rightarrow m_1 = 200\ln 2 \approx 139,\ m_2 = 400\ln 2 \approx 277.$$

If we add these allocations to the 69 monitoring units already allocated then we exceed the budget of 400. Thus, no monitoring effort should be allocated to patches 3 and 4, and the 331 monitoring units are distributed between patches 1 and 2 so that

$$\frac{\partial f_1}{\partial (m_1+69)} = \frac{\partial f_2}{\partial m_2} \text{ and } m_1 + m_2 = 331.$$

The solution to this equation is $m_1 = (400 – 100 \ln 2)/3 \approx 110$, $m_2 = (800 – 200 \ln 2)/3 \approx 220$. Combined with the 69 monitoring units already allocated to patch 1, there is one monitoring unit remaining and we add it to patch 1 (since monitoring effectiveness in higher there it will have the greatest impact). Thus, the optimal monitoring allocation to the four patches is

$$m_1^* = 69 + 110 + 1 = 180,\ m_2^* = 220,\ m_3^* = 0,\ m_4^* = 0.$$

That is, only the two patches with a high probability of pest presence are monitored. Even though patch 1 is prioritised initially, ultimately more monitoring effort is allocated to patch 2. This is because patch 2 has a larger area and so more monitoring units are required to adequately span the area.

Next we consider uniform distribution of monitoring effort over space. We would have 400/(1 + 2 + 2 + 1) = 66.67 monitoring units per unit area and

$$m_1^U = 67,\ m_2^U = 133,\ m_3^U = 133,\ m_4^U = 67.$$

If c_U = 100000 and c_D = 1000 then the total expecting saving made by allocating monitoring resources optimally is

$$\begin{aligned} T_1(\mathbf{m}^U) - T_1(\mathbf{m}^*) &= (c_U - c_D)\sum_{i=1}^{n} p_i \left(e^{-\lambda_i m_i^U} - e^{-\lambda_i m_i^*}\right) \\ &= (100000 - 1000)\begin{bmatrix} 0.04\left(e^{-0.01\times 67} - e^{-0.01\times 180}\right) + 0.04\left(e^{-0.005\times 133} - e^{-0.005\times 220}\right) \\ +0.005\left(e^{-0.005\times 133} - e^{-0.005\times 0}\right) + 0.005\left(e^{-0.01\times 67} - e^{-0.01\times 0}\right) \end{bmatrix} \\ &\approx 1612. \end{aligned}$$

That is, a manager saves roughly 1600 monetary units by targeting monitoring effort to patches 1 and 2 rather than applying it uniformly across space.

6.2. Using sighting records to determine when to declare eradication of an invasive species

Manuscript in preparation by Tracy Rout and Michael McCarthy

Methods

The probability that a species is extant given its sighting record can be calculated using the equation described in Solow (1993a). If the species is seen n times during a period of observation 0 to t, and then not seen during a further period of observation t_0, the probability that the species is extant is:

$$p(\text{extant}) = \frac{1}{1 + \frac{(1-\pi)\left(\left(\frac{t+t_0}{t}\right)^{n-1} - 1\right)}{\pi(n-1)}}, \qquad (1)$$

where π is the prior probability the species is extant, independent of sighting data (modified from Solow (1993a)).

Rule of thumb

The net economic cost of stopping after an absent survey is the cost of surveying, plus the expected cost of escape and damage if the species was present but went undetected (Regan et al. 2006). In Regan et al. (2006), the net expected cost (NEC) of stopping after t_0 absent surveys is defined as:

$$NEC_{t_0} = (t_0 - 1)C_s + C_e\left[p(1-q)\right]^{t_0},$$

where C_s is the cost of one survey, C_e is the expected cost of escape and damage, p is the probability that the species remains present, and q is the probability of detecting the species given that it is present. The expression $\left[p(1-q)\right]^{t_0}$ is the probability that the species was present but not detected for t_0 years, which can be substituted with Solow's equation to give:

$$NEC_{t_0} = (t_0 - 1)C_s + \frac{C_e}{1 + \frac{(1-\pi)\left(\left(\frac{t+t_0}{t}\right)^{n-1} - 1\right)}{\pi(n-1)}}. \qquad (2)$$

We can find the minimum NEC by setting the derivative of this equation to 0. There is no closed form expression for t_0, so we cannot derive a rule of thumb for the optimal value of t_0 that minimises the NEC. We can however identify a critical value of R, the ratio between the cost of surveying and the cost of escape ($R = C_s/C_e$). This critical value is the value of R at which it becomes optimal to stop surveying, i.e. for $R < R_{\text{crit}}$ we should keep surveying, while for $R > R_{\text{crit}}$ we should stop. We can express R_{crit} as:

$$R_{\text{crit}} = \frac{(n-1)^2(1-\pi)\pi y^n}{\left[(1-\pi)y^n - (1-n\pi)y\right]^2}$$

where $y = (t+t_0)/t$. R_{crit} can be approximated by a power function of y ($R_{\text{crit}} = ay^b$), which could give us a simpler rule of thumb.

Stochastic dynamic programming
Equation 2 does not include the possibility that the species may be seen in a future survey, incurring further costs of surveying and possible escape and damage. To incorporate these future expected costs, we can use stochastic dynamic programming, or SDP. SDP is an optimisation algorithm that can be applied to any system with a finite number of states, where sequential decisions must be made (Bellman 1957, Mangel and Clark 1988, Lubow 1996). SDP works backward over time, finding optimal decisions for each possible management scenario that take into account future expected costs (Bellman 1957, Mangel and Clark 1988, Lubow 1996).

The formulation of our SDP is similar to that in Regan et al. (2006). In each time step m (1 to M) there are two possible management decisions: to survey or to stop. The optimal decision is the one with the lowest expected cost. As outlined previously, the species has a sighting record in which it is seen n times over period t, and then not seen for period t_0. The optimal stopping time for particular values of n and t is the smallest t_0 where the expected cost of stopping is less than the expected cost of surveying. The expected cost of stopping is the probability that the species is extant given its sighting record, multiplied by the expected cost of escape and damage:

$$E_{stop}(m, t_0, n, t) = p(\text{species extant} \mid n, t, t_0)C_e,$$

which, substituting Solow's equation (eqn. 1), becomes:

$$E_{stop}(m, t_0, n, t) = \frac{C_e}{1 + \dfrac{(1-\pi)\left(\left(\dfrac{t+t_0}{t}\right)^{n-1} - 1\right)}{\pi(n-1)}}.$$

The expected cost of surveying must encompass two possibilities: the species is detected or not detected. The sighting record can be updated for each case. If the species is detected, the number of sightings n becomes $n + 1$, while the length of sighting period t becomes $t + t_0$. The period of absent surveys t_0 becomes 0. If the species is not detected, t_0 becomes $t_0 + 1$, while n and t remain constant. The expected cost of surveying is thus:

$$\begin{aligned} E_{\text{survey}}(m, t_0, n, t) &= C_s + p(\text{extant})p(\text{detected})E_{\text{opt}}(m+1, 0, n+1, t+t_0) + \\ &\quad (1 - p(\text{extant})p(\text{detected}))E_{\text{opt}}(m+1, t_0+1, n, t) \quad (m < M) \\ &= C_s \qquad (m = M). \end{aligned}$$

The probability that the species is extant is given by Solow's equation (1), while the probability that the species is detected can be estimated as n/t. E_{opt} is the expected cost of future optimal decisions, where the optimal decision gives the lowest expected cost:

$$E_{\text{opt}}(m, t_0, n, t) = \min[E_{\text{stop}}(m, t_0, n, t), E_{\text{survey}}(m, t_0, n, t)].$$

Case study
We apply the method to the example used in Regan et al.(2006): *Helenium amarum* in Queensland, Australia. *H. amarum* or bitterweed is toxic to stock, and if ingested causes vomiting, diarrhoea, and production of bitter undrinkable milk. It was first found in Queensland in 1953, and an eradication program began in the same year. After three years of herbicide and manual removal, only isolated patches of plants remained. Plants were not detected in several surveys between 1959 and 1987, but were detected in subsequent surveys. Between 1988 and 1992 no plants were detected, and the weed was declared eradicated (Tomley & Panetta 2002, as cited in Regan et al. 2006). We used the best estimate parameters in Regan et al. (2006), and the raw sighting data of *H. amarum* to parameterise the

models. The period of observation t is the period 1953 – 1987 (t = 34), and *H. amarum* was seen in thirty-two of those years (n = 32). We set the probability π to 0.5, and in the SDP we optimise the decisions over a time period of twenty years (M = 20).

Results

Rule of thumb

Using equation 2, the optimal stopping time for *H. amarum* is 11 years (Figure 1).

Stochastic dynamic programming

The SDP solution also finds an optimal stopping time of 11 years for *H. amarum* (Figure 2). The SDP produces optimal decisions for every possible sighting record (every combination of n, t and t_0). The results for t_0 = 11 are the first that find it optimal to stop surveying for any combination of n and t. The proportion of possible combinations of n and t for which it is optimal to stop increases as t_0 increases. By t_0 = 20 (Figure 3), the proportion for which it is optimal to stop is around 2/3 of the total number of combinations.

Discussion

The optimal stopping time of 11 years for *H. amarum*, from both our rule of thumb and SDP solution, is much more conservative than the optimal stopping time found by Regan et al. (2006). Regan et al. (2006) found an optimal stopping time of 3 years with the rule of thumb, and although they do not explicitly state the result from the SDP, they found the rule of thumb to be a good approximation of the SDP solution. The only element of our framework that differs from Regan et al. (2006) is the way the probability that the weed is extant is calculated. Solow’s equation produces much higher values for this probability than either the SDP or the rule of thumb methods described in Regan et al. (2006) (Figure 4a). This is made more visible when viewed on a logarithmic scale (Figure 4b). These higher probabilities translate into a more conservative stopping time.

Future Work

- Find an expression to approximate R_{crit}, which can be used as a rule of thumb.
- Compare the performance of the rule of thumb with the exact optimal results from the SDP.
- Develop and implement a more accurate method of estimating detectability from sighting data. The current equation used (= n/t) does not take into account the possibility that the species may still be extant during the period t_0, and so may overestimate detectability.
- Modify Solow’s equation for use with frequency data, as in Burgman et al. (1995). This could be more accurately applied to the *Helenium amarum* sighting data, which has several surveys per year in some years.
- Modify Solow’s equation for a declining population, as in Solow (1993b).
- Substitute Solow’s equation with other methods that are sensitive to the pattern of absences in the sighting data, as in Burgman et al. (1995) and McCarthy (1998).
- Apply the framework to monitoring a species of conservation concern.

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Figures

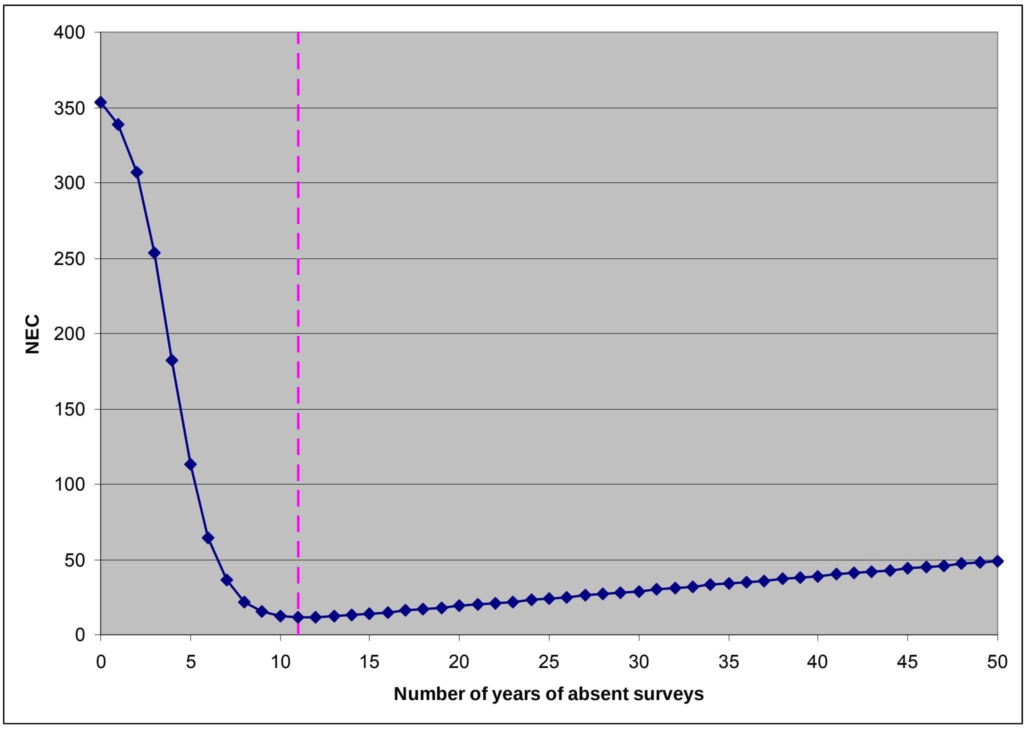

Figure 1: The net expected cost (NEC) as a function of the number of absent surveys using equation 2. The lowest NEC occurs after 11 years of absent surveys (marked with dotted line), making this the optimal stopping time.

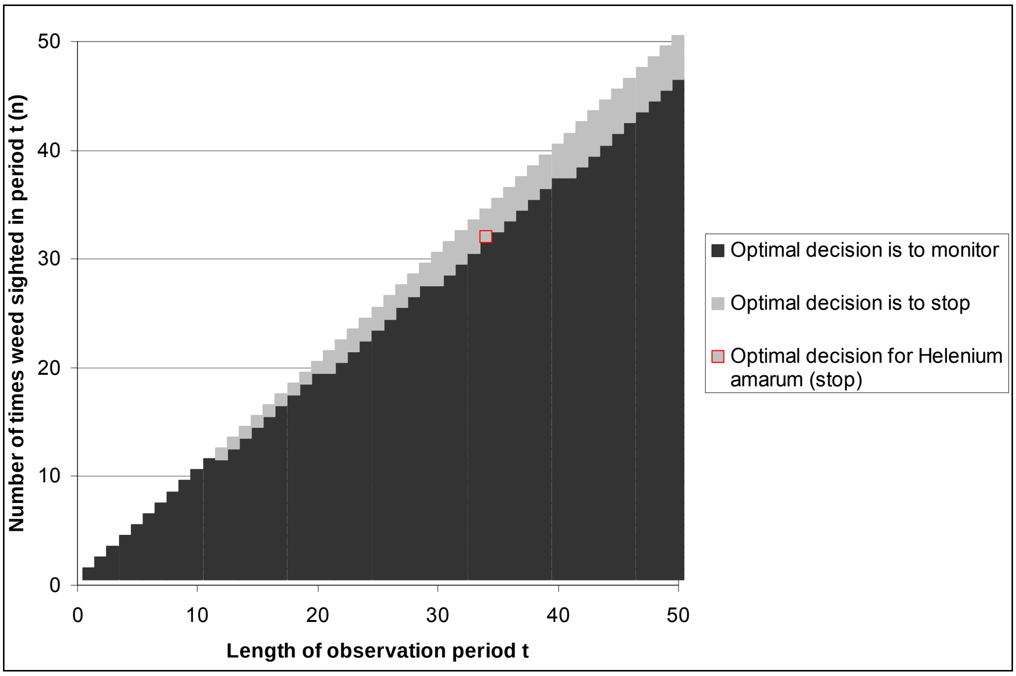

Figure 2: Optimal decisions after 11 years of absent surveys ($t_0 = 11$)

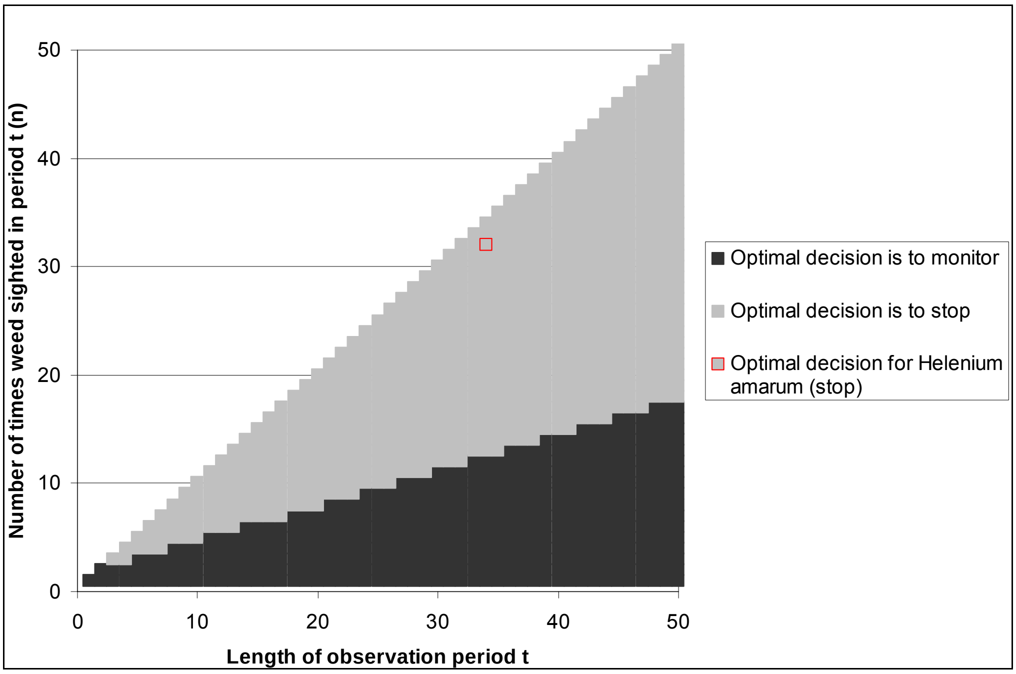

Figure 3: Optimal decisions after 20 years of absent surveys ($t_0 = 20$)

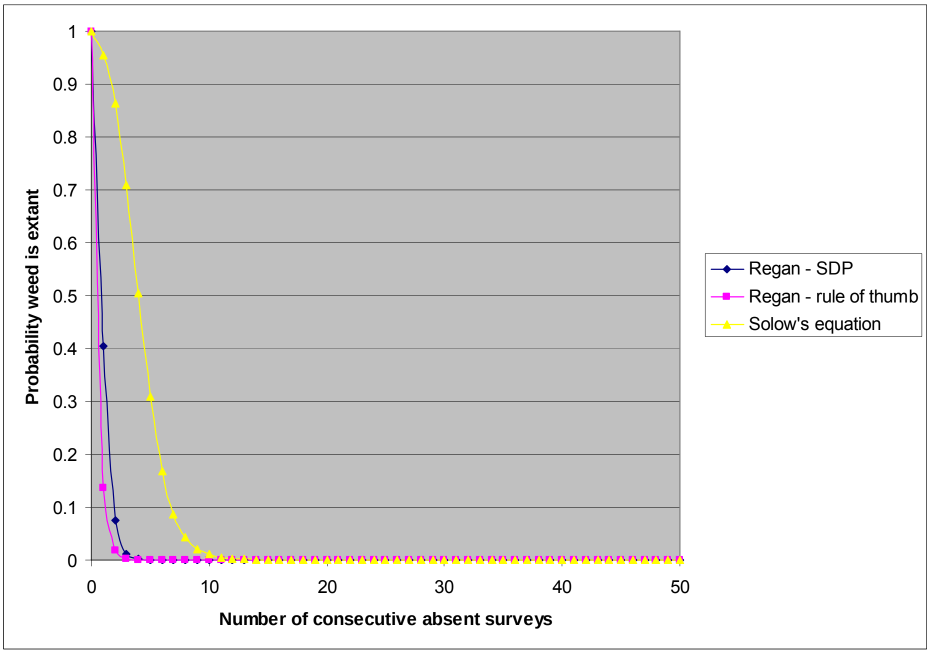

Figure 4a: The probability that the weed is extant as a function of the number of consecutive absent surveys, calculated with data for *H. amarum* using three different methods. 'Regan – SDP' and 'Regan – rule of thumb' were calculated using the methods described in Regan et al. (2006), and the best estimate parameters for *H. amarum* listed in Regan et al. (2006).

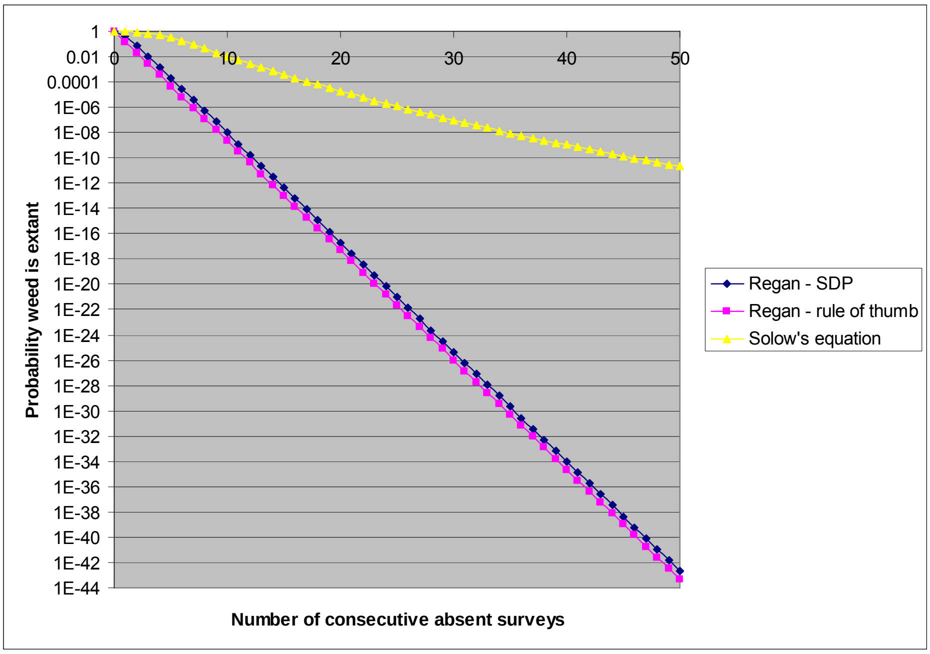

Figure 4b: As for Figure 4a, but shown on a logarithmic scale.

6.3. Draft manuscript presented in the previous ACERA report

Robust decisions about eradication of invasive species

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Running title: Robust eradication of invasive species

Abstract
Eradication of invasive species is uncertain because detecting them at low densities is often difficult. Decisions about when to declare that an invasive species has been eradicated need to balance the cost of additional surveys, the cost of the species escaping if eradication is declared when the species is still in fact present, and the probability that the species is still present. Costs may be estimated with some reliability, but it is difficult to estimate the probability that the species is present when it has not been observed for a particular period of time. In such circumstances, managers may wish to make decisions that are as robust as possible to errors in the estimate of this probability. We use info-gap decision theory to determine robust decision rules about the eradication of an invasive species, showing that a previously-derived result is not robust to uncertainty, but provides a lower limit on the number of surveys that should be conducted before declaring that an invasive species has been eradicated.

1. Introduction

Invasive species are one of the main threats to the world's biodiversity (Primack, 2006). In numerous parts of the world exotic predators have contributed to declines of native species, and weed invasion threatens native plant species and communities (Hunter, 2002; Primack, 2006). Because control and eradication of exotic species is often difficult once they have established, eradication of newly invaded species is desirable when possible.

It can be difficult to be sure that a species has been eradicated, because their detection is uncertain, especially when population sizes are small (Usher, 1989; Reed, 1996; Regan et al., 2006). How much effort should be spent on surveying for an invasive species if eradication is uncertain? If we declare that a species has been eradicated when it is present and stop conducting surveys for it, then we run the risk of it escaping and incurring a potentially large economic and environmental cost. However, if we continue surveying when it is already eradicated then we are spending money on surveys unnecessarily. Regan et al. (2006) balance these opposing risks by stopping to look for the invasive species when the total expected cost is minimised.

However, management decisions that maximise an expected outcome may not be robust to uncertainty (Ben-Haim, 2006). Importantly, such decisions can depend critically on the manager using an accurate model of the system being managed, but such models are often very uncertain. The model used by Regan et al. (2006) has two important parameters that are likely to be at least somewhat uncertain; the probability that the species remains present at a site from one year to the next and the probability of detecting the species when it is present. Sometimes these parameters can be estimated probabilistically (e.g., Wintle et al., 2004), but often bounds on their values are the best that can be achieved (Regan et al., 2006). These two parameters combine to provide r, the probability that a species remains at the site from one year to the next but is undetected in a survey (Regan et al., 2006).

Info-gap decision theory is designed to assist decision making in cases where there is considerable uncertainty about the model of the system (Ben-Haim, 2006). Rather than trying to optimize an expected outcome, info-gap methods find the management decision that is most robust to uncertainty while at the same time achieving a satisfactory outcome (e.g., Halpern et al., 2006). In this paper we determine management decisions about the eradication of an invasive species that are robust to uncertainty, basing the analysis on that of Regan et al. (2006).

2. The info-gap solution

Regan et al. (2006) found the number of absence surveys (n) that minimized the total expected cost

$$E(n) = nC_s + p(n)C_e, \qquad (1)$$

where C_s is the cost of surveying a site once, C_e is the expected cost of the invasive species escaping, and $p(n)$ is the probability that the species is still present if it has not been seen for the previous n surveys.

However, $p(n)$ is subject to considerable uncertainty, and ignoring this uncertainty may lead to suboptimal solutions. Instead of minimizing $E(n)$, the info-gap approach imposes the requirement that the expected cost should be no more than some acceptable upper bound (E_c), and aims to find the value of n that permits us to be as wrong as possible about $p(n)$ but still satisfy our constraint $E(n) \leq E_c$.

The info-gap solution begins with Regan et al.'s (2006) rule of thumb form for the probability that the invasive species is eradicated as our best guess for $p(n)$

$$\tilde{p}(n) = r^n \qquad (2)$$

For this model the robust optimal solution is to stop surveying if the invasive species has not been seen after n^* consecutive surveys, which is given by (see the appendix for details of the solution)

$$n^* = E_c/C_s + 1/\ln r. \qquad (3)$$

Superficially, this bears no resemblance to the rule obtained by Regan et al. (2006). Most notably, Eq. (3) does not depend on the expected cost of escape C_e, which was a critical parameter in the result of Regan et al. (2006).

However, a close correspondence can be seen between the result of Regan et al. (2006) and Eq. (3) by noting that the latter is predicated on the assumption that the robustness is non-negative, which leads to the constraint

$$n^* \geq \ln(-\frac{C_s}{C_e \ln r}) / \ln r\ . \qquad (4)$$

The right hand side of the inequality (4) is the solution obtained by Regan et al. (2006), so their solution provides a lower bound on the robust optimal solution Eq. (3). It can also be noted that the solution of Regan et al. (2006) has zero robustness to uncertainty, a general result of info-gap methods.

For the example used by Regan et al. (2006), $r = 0.136$ and $C_s/C_e = 1/350$. This imposes a lower bound on n^* of 3.28, and a lower limit of 3.78 on the ratio E_c/C_s. In this example r is sufficiently small such that Eq. (4) gives $n^* \approx E_c/C_s$ ($1/\ln(r) \approx -0.5$ is sufficiently small that it can be ignored), making the optimal expected total cost of monitoring after the last observation of the invasive species ($C_s n^*$) approximately equal to the total cost that is deemed to be satisfactory (E_c). This is an intuitive result;

when an invasive species is not observed we should keep surveying up until the total cost of the surveys becomes unsatisfactorily large.

If managers are sure that the model analyzed by Regan et al. (2006) accurately predicts the probability of detecting an invasive species, then they should use the solution of Regan et al. to minimize the total expected cost. However, if they are uncertain about its reliability and wish to minimize the chance of unacceptably large costs, then they should conduct at least as many surveys as the number suggested by the solution of Regan et al. (2006), with the robust-optimal number given by Eq. (3).

Acknowledgements

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Appendix

The info-gap approach to determining optimal monitoring of eradication of invasive species

We begin by defining:
$p(n)$ = probability that an invasive species is present after n surveys have been conducted during which the invasive species has remained undetected,
C_e = the expected cost of the invasive species escaping if surveys are stopped,
C_s = the cost of monitoring a site.
The total expected cost of monitoring n sites is then

$$E(n) = nC_s + C_e p(n). \tag{A1}$$

Since $p(n)$ is subject to significant uncertainty minimizing $E(n)$ for some assumed form for $p(n)$ may lead to suboptimal solutions. Instead we impose the performance requirement

$$E(n) \leq E_c, \tag{A2}$$

and aim to satisfice (A2) for all $p(n)$ in some uncertainty model such as the error bound model

$$U(p, \alpha) = \{\, p \big| \, | \, p(n) - \tilde{p}(n) \, | \leq \alpha \tilde{p}(n) \quad 0 \leq p(n) \leq 1 \,\}, \ \alpha \geq 0 \,, \tag{A3}$$

with the largest horizon of uncertainty (the robustness)

$$\hat{\alpha}(n) = \max\{\alpha \mid (\max_{p \in U} E(n)) \leq E_c\}. \tag{A4}$$

From (A1) and (A3)

$$\max_{p \in U} E(n) = nC_s + C_e(1+\alpha)\ \tilde{p}(n), \tag{A5}$$

(assuming $(1+\alpha)\ \tilde{p}(n) \leq 1$) and hence from (A4) (equating the rhs of (A5) to E_c)

$$\hat{\alpha}(n) = \begin{cases} [(E_c - nC_s) \,/\, C_e \tilde{p}(n)] - 1 & \text{when} \geq 0 \\ 0 & \text{else.} \end{cases} \tag{A6}$$

Following Regan et al. (2006) we take the "rule of thumb" form

$$\tilde{p}(n) = [p(1-q)]^n, \tag{A7}$$

where
p = probability that the invasive species persists from one year to the next
q = probability that the species is detected given that it is present
$r = p(1 - q)$.
In an info-gap setting the robust-optimal value for the decision variable n is obtained by maximizing the robustness (A6). Assuming $\hat{\alpha}(n) > 0$, this is equivalent to maximizing

$$f(n) = (E_c - nC_s) \,/\, \tilde{p}(n). \tag{A8}$$

Differentiating (A8) with respect to n we obtain

$$\frac{df}{dn} = -\frac{C_s}{\tilde{p}(n)} - \frac{E_c - nC_s}{[\tilde{p}(n)]^2}\frac{d\tilde{p}}{dn}$$

$$= \frac{1}{\tilde{p}(n)}\{-(E_c - nC_m)\frac{d}{dn}[\ln \tilde{p}(n)] - C_s\}, \tag{A9}$$

which holds for general $\tilde{p}(n)$. For the case (A7)

$$\frac{d}{dn}[\ln \tilde{p}(n)] = \ln r \,, \tag{A10}$$

so that on setting *df/dn* = 0 we obtain from (A9) and (A10) the robust optimal solution

$$n^* = \frac{E_c}{C_s} + \frac{1}{\ln r}\,, \tag{A11}$$

which seemingly bears no relation to the rule of thumb found by Regan et al. (2006). The above, however, is predicated on the assumption that $\hat{\alpha}(n) \geq 0$. Substituting (A11) into (A6) we thus see that n^* is the robust optimal solution provided

$$\hat{\alpha}(n) = -\frac{C_s}{C_e \tilde{p}(n) \ln r} - 1 \geq 0 \;\; n^*. \tag{A12}$$

For the particular case (A7), (A11) implies that

$$n^* \geq \ln[-\frac{C_s}{C_e \ln r}] / \ln r \,. \tag{A13}$$

That is, the rule-of-thumb solution (the rhs of (A13)) is a lower bound on the robust optimal solution (A11). It is also seen from the above that the rule-of-thumb solution has zero robustness to uncertainty. The above results can be used in a number of ways. For example (A11) and (A13) imply

$$\frac{E_c}{C_s} \geq -\frac{1}{\ln r}[1 - \ln(-\frac{C_s}{C_e \ln r})]\,, \tag{A14}$$

which places a lower bound on cost aspirations (E_c) relative to monitoring costs.

For example, from Regan et al. (2006) if we take $p = 0.8$, $q = 0.83$ (i.e., $r = 0.136$) and $C_s/C_e = 1/350$, (A14) implies that

$$\frac{E_c}{C_s} \geq 3.78\,, \tag{A15}$$

which places a lower bound, from (A11), of 3.28 on the robust optimal solution n^*.

Also, instead of maximizing $\hat{\alpha}(n)$ one could use (A6) to determine trade-off values for n depending on desired levels of robustness and/or aspirations (E_c).