Report Cover Page

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Title
Simplifying models of alien biota
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Report #1

Summary

This project will develop methods that simplify complex spatial dynamic models of weed and pest spread. The motivation for the project is to provide a way of simplifying predictions of spatial population dynamics, to enable them to be linked in an efficient manner to the on-going work on statistical spatial predictions.

The project started less than two months ago, three months behind the originally envisaged schedule, slowed by contract formalities and acquiring the right expertise for the postdoctoral fellowship. Nevertheless, the first deliverable was achieved on time. This involved the submission of a publication to MODSIM07. In this publication a method for the sensitivity analysis of functions in a model was tested on a simple systems model.

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ACERA Project No. 07/04

Simplifying models of alien biota

John Hearne; RMIT

Report #1; 31 July, 2007

Background

This project was initially scheduled to begin on 28-Feb-07. Due to a number of delays, including ACERA's legal agents, it was not possible to begin the project until 4 June, 2007. On this date the postdoctoral fellow associated with the project commenced work. Thus the project started three months behind schedule.

Despite the delay, the main priority has been to honour the original deliverable deadline of 31 July, 2007. This deliverable required the submission of a paper on: 'A technique for the sensitivity analysis of functions in relation to decision-making objectives'. This has been achieved. The manuscript is attached here.

Method

The methods outlined in the proposal indicated that an optimal control approach would be tried first to solve the problem of simplifying complex dynamic models. It was also stated that a reformulation of the problem as an optimisation problem with parameterised functions should work. After some initial investigation with the first method we concluded that there was not enough time to meet the deadline with this approach. The second approach was then taken. A method of changing the shape of a function was proposed and tested on a simple model that involved a decision criterion.

The approach is promising and needs to be tested now on a large complex model. Initial discussions with Dr Darren Kriticos have been held with a view to using his model. Now that the first deliverable has been achieved, work will commence on this very soon.

Problems

There was some difficulty finding a postdoctoral fellow with the appropriate experience for this project. The position was advertised widely. The first choice applicant later withdrew his application. The eventual appointee has strong skills in Applied Mathematics but has had some trouble adjusting to ecosystem models at the interface between science and management. This has been a real concern when trying to meet the extremely tight deadline imposed by the schedule for the first deliverable.

A Technique for the Sensitivity Analysis of Functions in Relation to Decision-making Objectives

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EXTENDED ABSTRACT

Sensitivity analysis is usually focussed on parameters and is a relatively well-developed field compared with function sensitivity. But how sensitive are model conclusions to the choice of functions used in the right hand side of differential equation models? Most work in this area has been scenario-based where alternative functions are tested. In this paper, we examine the sensitivity of a model to changes in the shape of the functions. We do this in an automated way without the need to specify alternative functional forms.

The question then is how much can we change a function that defines the dynamics of the system without producing a significant change in some performance measure? If the changes in functions need to be large in some sense, to cause significant changes in a performance measure, then there is less need to focus attention on getting the model functions correct. A method of approach to this type of analysis is presented and illustrated on an ecosystem model. Testing the proposed method on a simple model demonstrates that quite large changes can be made to functions before reaching a critical value in the decision criterion. This insight is as useful as the corresponding knowledge of the effect of uncertainty in parameter values.

1. INTRODUCTION

It has long been recognised that, "the simple obtaining of solutions for the equations of the mathematical model of a dynamic system – or even a set of solutions – is no longer sufficient" (Tomovic 1963). Further, in 1968, Quade expressed the view that: "A good system study will include sensitivity tests on the assumptions in order to find out which ones really affect the outcome and to what extent. This enables the analyst to determine where further investigation of assumptions is needed". Along these lines Forrester (1969) demonstrated that the question of sensitivity is important from a policy viewpoint only when parameter changes would render a proposed policy ineffective. At that time and subsequently most sensitivity analyses were only performed on parameters and initial values. This approach can be found, for example, in Barnes and Yeaple (1968), Thornton and Lessem (1976), and Vermeulen and De Jongh (1976 and 1977).

A notable early exception to straight parameter sensitivity analysis was the practical approach adopted by Ford and Gardiner (1979). They convened a workshop of public and private leaders where the group was presented with model forecasts and asked to decide on a policy. Changes were then made to the model and the group was presented with the new forecasts. Based on these forecasts, the group was again asked to vote on the policy. If the policy decision was unchanged the model could be regarded as insensitive to the changes. So it might well be that some change in a parameter value or function causes a very large change in a state variable but if this does not alter the decision of the policy-making body then in practical terms the model is insensitive.

The main purpose of a sensitivity analysis of a model used in decision support should be to determine the extent to which the decisions or policies based on model results are robust with respect to the uncertainty in the model. Walker *et al* (2003) recently noted the increasing requirement to articulate uncertainty, when working at the interface of science and management, in model-based decision support. They recognise two extremes as a feature of the nature of uncertainty: These are 'epistemic uncertainty', which is due to the imperfection of our knowledge and may be reduced by more research or data, and 'variability uncertainty, which is due to the inherent variability in a system.

Epistemic uncertainty includes uncertainty in parameter values, model inputs, and the functions in a model. As noted, most analyses of models do include a sensitivity analysis (SA) of the parameters and a scenario analysis of the inputs. Alternate functions within a model, however, are only sometimes tested and in an *ad hoc* manner. Little has been done to perform a SA of *functions* in the automated way that is done with parameters. This paper takes some tentative steps towards addressing this problem.

2. METHOD

Consider the following system of difference equations:

$$x_i(t+\Delta t) = x_i(t) + \Delta t\, f_i(\underline{x}, t, \underline{\alpha}) \quad i = 1,2,\ldots,n \tag{1}$$

where $\underline{x}$ is the state vector and $\underline{\alpha}$ a vector of parameters.

A basic parameter sensitivity analysis involves changing the values of the parameters in the functions $f_i(\underline{x}, t, \underline{\alpha})$ by a small amount, one at a time, and observing the change it produces in the output. Although changing the parameters in the function $f_i(\underline{x}, t, \underline{\alpha})$ does change the shape of the function it does so in very restrictive ways. Clearly other changes to the shape of the function are possible. This is an important consideration if there is uncertainty about the appropriateness of the functional form chosen for the model. The functions contain the information about the dynamics of the model, with different functional forms corresponding to different choices of dynamics for the system. Changes in the functions then correspond to changes in the dynamics of the model.

A possible pragmatic approach for this more general form of function SA is to multiply each function or rate by a parameter with a nominal value of one. These parameters can then be perturbed as is done in parameter SA. This should yield some indication as to which rates are the most sensitive. This method was tried with some success by Lawrie and Hearne (2007).
One of the shortcomings of this approach, however, is that no information is obtained on the sensitivity of the output to changes in the shape of the functions. The simplest approach towards this end, going beyond the method mentioned above, is to multiply each function by the following function which comprises a product of triangular-shaped functions:

$$\begin{aligned} h(\underline{x}, \underline{p}, \underline{m}) &= (1 + h_{x1}(x_1, p_{x1}, m_{x1})) \\ &\times (1 + h_{x2}(x_2, p_{x2}, m_{x2}))\ldots \\ &\times (1 + h_{xn}(x_n, p_{xn}, m_{xn})), \end{aligned} \tag{2}$$

where,

$$h_{xi}(x_i, p_{xi}, m_{xi}) = m_{xi}(x_i - c)/(p_{xi} - c)$$
$$\text{and } c = \begin{cases} a_i \text{ if } x \le p_{xi}, \\ b_i \text{ if } x > p_{xi}. \end{cases} \tag{3}$$

Note that by choosing m_{xi} to be negative we can invert the triangular shaped function h_{xi}.
Reasonable choices of a_i and b_i are the respective minimum and maximum values of the corresponding state variable over the solution interval.

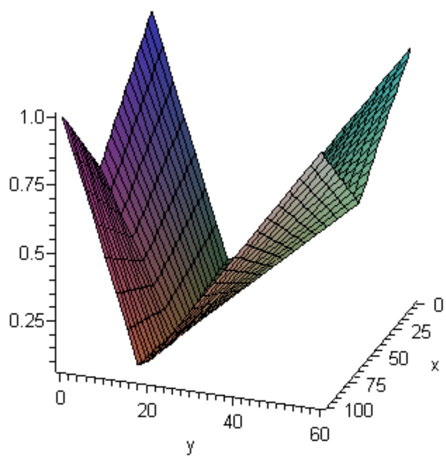

Figure 1. An example of the function $h(\underline{x}, \underline{p}, \underline{m})$ in two dimensions, where both m_{x1} and m_{x2} are negative.

The function $h(\underline{x}, \underline{p}, \underline{m})$ deviates from the constant function 1, where the greatest change occurs at $\underline{x} = \underline{p}$, with magnitude determined by $\underline{m}$.

The sensitivity analysis now proceeds by investigating changes in each of the functions

$f_i(\underline{x}, t, \underline{\alpha})$ obtained by multiplying each in turn by a function $h(\underline{x}, \underline{p}, \underline{m})$. The sensitivity analysis tests the sensitivity of some objective measure chosen by the user. In this project we are interested in a decision-making objective. In this context further development of our idea is best achieved with an illustrative example.

3. ILLUSTRATIVE EXAMPLE

An agricultural product X will be ready for harvesting in T (=12) months time. A pest species Y consumes X at a certain rate dependent on the density of X. The damage caused by Y is unacceptable and two means of controlling the pest have been proposed: (1) biological control through the introduction of a parasitoid Z and (2) chemical control. The first method is much cheaper and also more desirable from environmental considerations but there is more confidence in the efficacy of chemical control. To facilitate making a decision, a model of the system with Z has been formulated. The aim of the model is to answer the following question:

Will the introduction of population Z ensure that the biomass of X achieves a minimum level at harvest time T? In particular will the 10^{th} percentile of X be above a threshold value V (=60)?

Let x_1, x_2, and x_3 denote the population levels of X, Y, and Z, respectively. The model is given by the system of equations (1) with the following RHS functions:

$$f_1 = r_1 x_1 (1 - x_1 / k) - \frac{\alpha_1 x_2 x_1}{(x_1 + \alpha_2)},$$
$$f_2 = \frac{r_2 x_2 x_1}{(x_1 + \alpha_2)} - \frac{\beta_1 x_3 x_2}{(x_2 + \beta_2)},$$
$$f_3 = \frac{r_3 x_3 x_2}{(x_2 + \beta_2)} - \gamma x_3. \tag{4}$$

Initial and parameter values are

$$x_1(0) = 40,\ x_2(0) = 10,\ x_3(0) = 6,$$
$$r_1 = 0.8,\ r_2 = 0.4,\ r_3 = 0.25,$$
$$\alpha_1 = 0.4, \alpha_2 = 20, \beta_1 = 0.2, \beta_2 = 4,$$
$$\gamma = 0.001,\ k = 100(1 + N(0,10)), \tag{5}$$

where *N(0,10)* is a normally distributed random number with mean 0 and standard deviation 10.

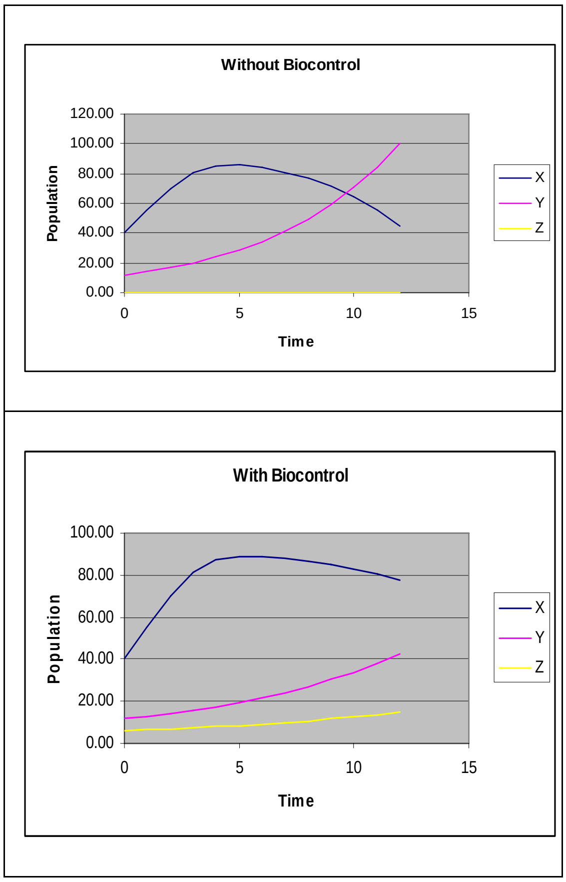

Figure 2. Deterministic solution: The system behaviour without bio-control shows population *X* decreasing below the acceptable threshold. The introduction of population *Z* reduces *Y* enabling *X* to maintain a level well above the threshold at harvest.

That the introduction of Z is effective can be seen by comparing the two graphs with and without population Z in Figure 2. These are solutions of the deterministic model with *k* held constant at 100. Further analysis was undertaken by performing 500 simulations of the stochastic model. These solutions indicated that at the final time T, X will have a mean of approximately 78 and a 10^{th} percentile of 66 (>*V*). This suggests that the decision can be made tentatively to go for option (1), biological control.

Normally at this point sensitivity analysis of parameters and initial values would be undertaken and possibly some experimenting with alternative model formulations. As the purpose of this project is to go beyond that, we assume that all parameter and initial values are perfectly known.

The question then remains whether the functions $f_i(\underline{x},t,\underline{\alpha})$ of the model are correct. In particular, we are interested in the following:

By how much can the functions be distorted while still ensuring that the decision criterion is satisfied? The decision criterion being that the 10th percentile of population X lies above the threshold V at the final time and hence that the first option for control will be the preferred one?

Function	$\lvert\underline{m}\rvert$	Relative Insensitivity	
f_1	0.97	6.01	
f_2	0.16	1.00	
f_3	2.30	14.35	
Function	Peak Vector		
	m_{x1}	m_{x2}	m_{x3}
f_1	-0.35	-0.9	
f_2	0.11	0.11	0.04
f_3		-1.5	-1.75
Function	Critical Point		
	p_{x1}	p_{x2}	p_{x3}
f_1	58	18	any z
f_2	56	28	8
f_3	any x	18	10

Table 1: Results applying the proposed method to the illustrative model. The third column, Relative Insensitivity, is indicative of the relative magnitude of change that can be made to a function before the critical value of the decision criterion is reached. The last three rows contain the point where the function is most sensitive to function changes.

If the criterion is satisfied, despite large changes to the functions, then one might conclude that the decision is insensitive to the choice of model functions. We now formulate the mathematical problem to answer this question.

Formulation of the function sensitivity problem

Problem P1

Consider a change to one function $f_i(\underline{x},t,\underline{\alpha})$ at a time, given by $h(\underline{x},\underline{p},\underline{m})\,f_i(\underline{x},t,\underline{\alpha})$. For the *i th* equation this means:

Find $(\underline{p}^*,\underline{m}^*)$, the solution to the constrained minimization problem:

$$\min_{\underline{p},\underline{m}} \underline{m}^2$$

constrained by the condition that 10th percentile $x_1(T) \le V$.

This is the smallest change of $f_i(\underline{x},t,\underline{\alpha})$ which no longer ensures that the harvest has minimum biomass greater than 60 units.

Effectively P1 means that we find the position in state space where the model is most sensitive to changes in the function $f_i(\underline{x},t,\underline{\alpha})$. Moreover we can determine if increasing the function or decreasing the function at that point produces the greatest change in the output measure – through the sign of $\underline{m}$. This means that regardless of position or direction (increase or decrease) any smaller change in the function will ensure that the final level of population X is acceptable, and hence robust to the decision.

4. RESULTS

Problem P1 was solved for the three cases corresponding to each of the three RHS functions. The results are shown in Table 1. Note that the f_2 function is the most sensitive while f_3 can endure much larger changes before the decision criterion is violated.

The position of the peak changes the shape of the function. This in turn generally will influence the results. The original function $f_1(\underline{x},t,\underline{\alpha})$ is shown in Figure 3. The Modified function $h(\underline{x},\underline{p}^*,\underline{m}^*)\,f_1(\underline{x},t,\underline{\alpha})$, where the $(\underline{p}^*,\underline{m}^*)$ values for function 1 are given in table 1, is shown in Figure 4.

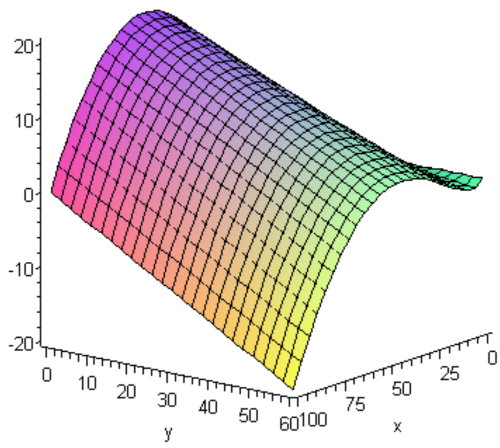

Figure 3. This figure is the original function $f_1(\underline{x}, t, \underline{\alpha})$ given in Equation 4 above.

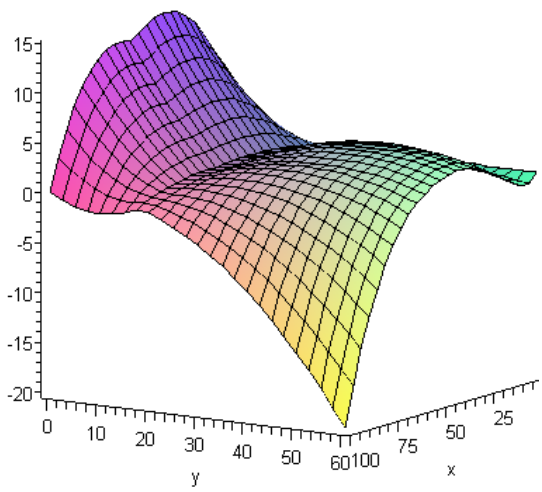

Figure 4. This figure shows the modified function $h(\underline{x}, \underline{p}^*, \underline{m}^*) f_1(\underline{x}, t, \underline{\alpha})$.

To further demonstrate this we change the function f_1 in the same way as recorded in Table 1. This time we locate the peak change at (65, 30, any z) and repeat the simulations. This produced a 10^{th} percentile for population X greater than 68 compared with 60 in the previous solution.

For comparison purposes with previous work each RHS functions was multiplied by a constant parameter of nominal value one. These were then perturbed by 1% one at a time and the change in the 10^{th} percentile of X at the final time noted. This yields changes of 2.36%, 1.89%, and 1.5%.

These results suggest that f_1 is the most sensitive followed by f_2 while Table 1 indicates the reverse. The difference is that the results in Table 1 tell us something about the relative importance of the shape of the functions.

5. CONCLUSION

Decision and policy making is determined or influenced by model output. In this context the techniques needed to explore the relationship between model output and uncertainties in parameter and initial values is well-developed. Uncertainty in the functions used in a model is less so. This is often dealt with on a trail and error basis or as a scenario analysis. This is difficult or too time-consuming to do with large complex models. In this paper we have attempted to investigate the effects of uncertainty in functions through an automated process.

Testing the proposed method on a simple model has demonstrated that quite large changes can be made to functions before reaching a critical value in the decision criterion. This insight is as useful as the corresponding knowledge of the effect of uncertainty in parameter values.

We plan to investigate the procedure further by following a similar process for each term (ie each rate) in the RHS function. This should in turn yield useful information about all the relationships within a model. The intention is then to test the whole procedure on a large complex model.

6. ACKNOWLEDGEMENTS

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