ACERA Project 1004

A tool to support the decision to switch between eradication and containment of an invasion

Switching Point Model
Version 2.0

User Manual

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ACERA Project 1007

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1. Introduction

The primary objective of this study is to develop a decision tool that will help decide whether a new eradication program is worth pursuing, or whether an existing program should continue. The decision model must be based on solid scientific and economic foundations and must be robust to uncertainty. To achieve these objectives the model must be based on a clear description of the state of the invasion at any given time and must consider the future consequences of current decisions, including the decision to do nothing. A secondary objective is to design the model to be as simple as possible, while ensuring it captures the key features of a managed biological invasion.

The first step in the project was to implement the simplest possible model that could be used by anyone with a basic understanding of the three components of a managed invasion:

- A growth (spread) curve
- A damage curve
- A cost curve

The model was implemented as an Excel file. The mathematical model underlying this work is described in the Appendix. Version 1.0 of the model required the user to enter five parameters:

- Specific growth rate (r)
- Area at risk (K in ha)
- Damage (β_D in \$/ha)
- Control Cost (β_C in \$/ha)
- Discount rate (δ in %)

Using this information the model calculates the maximum area that should be targeted for eradication. The calculation is based on minimising the combined cost of controlling the invasion plus the damages caused by the invasion. The point at which eradication is no longer the optimal course of action is referred to as the *Switching Point*.

The model was distributed to a group of potential end users in February 2011. The main comment received was that users would require some help coming up with values for the damage and cost parameters (β_D and β_C) and for the specific growth rate (r). Version 2.0 of the model addresses this request and should enable users to apply the model to case studies with some help from the research team. Application of the model to case studies will involve exhaustive sensitivity analyses to help characterise different invasion types and understand the role of uncertainty in the decision process.

This manual describes version 2.0 of the model and explains its use. The model is contained in the Excel file Switching_Point_Mod_v2.0.xls. In summary, the decision whether to eradicate is based on minimising the total cost of the invasion over time. This includes the cost of control as well as the damage caused by allowing the invasion to spread. Both the control cost and the damage cost are measured on a per hectare basis. This means that each additional hectare invaded increases all costs and damages proportionally, which is the same as assuming that the landscape is invaded in a homogeneous fashion.

2. Model Description

The Switching Point model consists of four worksheets: Model, SP, Growth and Info. All user interaction occurs within the Model worksheet (Figure 1). The remaining worksheets perform the calculations required to plot the switching point and growth graphs but cannot be modified by the user.

Dynamic Switching Point Model
Version 2.0, 13 March 2011
Data entry worksheet

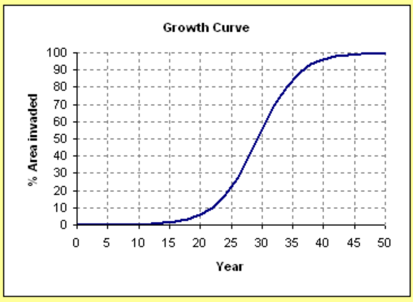
INVASION PARAMETERS	
Total area at risk (ha):	6,500
Specific growth rate	0.3

INVASION DAMAGES		$/unit	Units affected at saturation	Units	Total $ 181
1	Household damages	4	600	hh	
2	Costs of private control measures	200	600	hh	
3	Lost trade opportunities	120,000	1	Total	
4	Damage to human health	180	400	people	Area
5	The cost of missed work	260	400	people	Fraction
6	Losses in property values	45	2925	ha	0.45
7	Damages to livestock and agriculture	25	3575	ha	0.55
8	Damages to public property	5	1300	ha	0.20
9	Loss of recreational values	530	975	ha	0.15
10	Loss of biodiversity	5	2340	ha	0.36
11	Other				

CONTROL COSTS		Cost	Units	Total $ 1,704
1	Labour	850	$/ha	
2	Materials	460	$/ha	
3	Damage caused to industry	140	$/ha	
4	Damage caused to environment	230	$/ha	
5	Travel	35,000	$	
6	Administration	120,000	$	

ECONOMIC PARAMETERS	
Discount rate	7%
Total budget (PV)	1,500,000

RESULTS	ha	Total Cost
Switching point	3,640	6,202,667
MB=0 at	4,399	7,494,750
Max area treatable	880	1,500,000



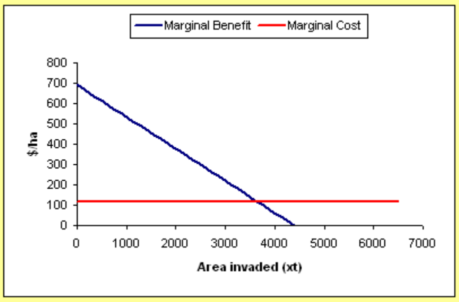


Figure 1. The Model Worksheet.

The Model worksheet contains 5 tables:

- **Invasion Parameters**: are associated with the growth curve represented by a logistic equation in the plot located on the top right in Figure 1.

- **Invasion Damages**: contains a breakdown of the various components of the damage curve. The parameters contained in this table are used to calculate a single value for damages per hectare. The cell labelled *Total* on the top right of this table represents the Damage Parameter (β_D).

- **Control Costs**: contains a breakdown of the various components of the cost curve. The parameters contained in this table are used to calculate as single value for the cost of control per hectare. The cell labelled *Total* on the top right of this table represents the Cost Parameter (β_C).

- **Economic Parameters**: this table comprises the discount rate and the budget. The budget does not affect the actual calculation of the switching point, but it is used to calculate the maximum area that could be eradicated when the budget constraint is binding.

- **Results**: this table includes the switching point, the point at which the marginal benefit of control is zero (MB=0) and the maximum area that can be treated with the given budget constraint. The costs associated with these three points are also presented. The plot to the right of this table presents the switching point graphically as the intersection between the marginal benefit (MB) and marginal cost (MC) curves (bottom right in Figure 1).

These components are explained in more detail below.

2.1 The Growth Curve

While recognising that the growth of an uncontrolled invasion is driven by two processes: *local growth* and *spatial spread*, here we use a simplified description of the invasion. The model considers the area invaded at any time and the total area at risk, but does not account for variations in population density through space. The state of the invasion is assumed to be fully described by area invaded at any given point in time (denoted as x_t).

The spread of the invasion is represented by a logistic equation (see top right section of Figure 1 and equation 4 the Appendix). The *Specific Growth Rate* (r) determines the speed at which the population grows and the carrying capacity parameter (K) represents the *total area at risk*. These parameter values can be estimated statistically if time-series data on area invaded are available. Alternatively the growth curve next to the Invasion Parameters table can be used to help set the value of r. The curve shows the percentage of the total area at risk that would be invaded over time if an invasion of one ha (at time zero) is allowed to spread. Suppose an expert panel estimates that 40% of the area at risk will be invaded within 20 years if the invasion is allowed to spread, with this information we can find the appropriate value for r through trial and error (r=0.42 results in x_t=40 at t=20 for the example in Table 1).

2.2 Damages

The damages per hectare are calculated from the itemised entries in the "Invasion Damage" table. Note that different parameters have different units of measurement and keep in mind that not all invasions will have entries for every parameter. This will depend on the type of invader and on the landscape being invaded. For example many weeds may not have health effects and/or cause people to miss work. Other invasions may not cause lost trade opportunities or damages to agriculture. The *units affected at saturation* refer to the total amount of each type of damage that would occur once the whole area at risk is invaded. The cost parameters ($/unit) must be compatible with their corresponding units. The list of possible damages is replicated below followed by brief explanations of each.

ID	Description	Units of measurement
D1	Household damages	number of households (hh)
D2	Costs of private control measures	number of households (hh)
D3	Lost trade opportunities	Total amount
D4	Damage to human health	number of people
D5	The cost of missed work	number of people
D6	Losses in property values	ha
D7	Damages to livestock and agriculture	ha
D8	Damages to public property	ha
D9	Loss of recreational values	ha
D10	Loss of biodiversity	ha
D11	Other	

Household damages (D1) and costs of private control measures (D2) are in terms of households likely to be affected. As an example assume there are 2,400 households in the area at risk and ¼ of them are expected to be affected. Then the entry under *Units affected at saturation* is $0.25 \times 2,400 = 600$ households. The dollar value for this damage would be the average cost incurred by households.

Lost trade opportunities (D3) are presented as a total amount in the table, but they could be expressed in different units depending on the nature of this loss. The values in Figure 1 imply that, if the whole area at risk is allowed to be invaded trade will decrease by $120,000 per year. This could occur, for example, if the number of tourists visiting the area drops as a result of the invasion. If there are several industries or sectors affected this parameter would measure the total loss across all industries or sectors.

Damage to human health (D4) and the cost of missed work (D5) are in terms of number of people affected. As an example, assume the population in the area at risk is 10,000 people and 4% of them are expected to be affected at saturation, then the entry in this column is $10,000 \times 0.04 = 400$ people. The cost per unit could be calculated based on average medical expenses per person for D4 and average numbers of hours missed multiplied by the average wage rate for D5.

The next five damage types (D6 to D10) are on a per-hectare basis. For these items there is an additional column that contains the area fraction that would be affected at saturation. This simplifies data entry by allowing potential damages to be expressed relative to the total area at risk. For example, if a particular damage applies only to $1/5^{th}$ of the area at risk, the actual area affected at saturation would be $0.2 \times 6,500 = 1,300$ ha. The actual areas (in ha) are calculated automatically from the area at risk and the given fractions, the cells associated with these variables are shaded in grey to indicate they are not to be modified by the user. Now let's look at the individual costs per hectare.

Losses in property values (D6) are likely to depend on the type of land affected. Some invasions may affect rural land prices but not urban prices for example. Or different land types may be affected at different rates. In such cases the parameter would be a weighed average based on the areas of different land types and their respective loss in value.

Losses to livestock and agriculture (D7) are normally straightforward to calculate if we know the types of agricultural enterprises, their net revenues per ha, and the expected reduction in revenues caused by the invader. When several types of enterprises are affected these entries would be weighed averages.

Damages to public property (D8) may overlap with losses in public land values under D6 and care must be taken not to double count. As an example, damages to roads could be measured as the cost of repairing deterioration caused by the invader.

Loss of recreational values (D9) and loss of biodiversity (D10) may be difficult to measure. These values are normally calculated using stated-preference methods, based on surveys that may be time consuming and expensive to apply. In the case of recreational values, care must be taken not to double count if trade losses to the tourism sector have already been included in D3.

Other costs not considered on this list can be included in the final row of the damages table (D11).

2.3 Control Costs

The costs per ha are calculated from the itemised entries in the "Control Costs" table. As above, different parameters have different units of measurement and not all types of costs apply to all invasions. If it takes several years to clear the invader from an area, costs should be entered as present values. For example, if clearing weeds from one ha of land requires 10 years of repeat visits until the seedbank is exhausted, then the cost entries must consider this.

The list of control costs is replicated below followed by brief explanations of each.

ID	Description	Units
C1	Labour	$/ha
C2	Materials	$/ha
C3	Damage caused to industry	$/ha
C4	Damage caused to environment	$/ha
C5	Travel	$
C6	Administration	$

Labour costs (C1) are calculated as the number of hours required to clear one hectare (including search and treatment activities) multiplied by the wage rate. If several years of monitoring and control are required the total discounted costs must be calculated.

Materials costs (C2) normally represent chemicals used for treatment, but could also include depreciation of any machinery and equipment used. As above, present values should be used when repeat visits are required.

The costs of damages to industry (C3) or the environment (C4) are those caused by the control method itself, as opposed to those caused by the invading organism considered in the previous section. For example, damages to industry are caused when control involves killing livestock, and damages to the environment occur when chemicals used to control the invader also kill native organisms.

The costs of travel (C5) and administration (C6) are calculated on a per-year rather than per-hectare basis. Cost of travel to infested sites is the sum of air and road travel plus any per-diems paid to staff. The costs of administration would include office expenses, including annual salaries of managers but excluding the salaries associated with labour already counted in D1.

2.4 Results

The results are shown as a graph that displays the marginal benefit (MB) and marginal cost (MC) of eradication. The intersection between the two lines indicates the point at which it is optimal to end the eradication program and consider whether to attempt containment or abandon the control effort.

The box labelled *Switching point* within the results table indicates the area at which this intersection occurs. When the area invaded is below this value, it is optimal to continue with the eradication attempt, when it is above this value, eradication is not economically optimal and the containment option should be considered. The results table also shows the point at which the marginal benefit curve crosses the horizontal axis (MB=0), and the maximum area that could be treated at a given budget. In many cases we find that eradication is not possible with the given budget even though it is optimal to eradicate (i.e. the actual area invaded is below the switching point).

The model can be applied to any invasion by filling in the entries within white cells in Figure 1, all other entries are protected and cannot be changed.

3. Appendix: Mathematical Model

Consider the case of a managed invasion of size x_t at time t and a control that can reduce the invasion by an amount u_t per unit applied. The damage (D) caused by the invasion depends on its size, and the cost (C) of controlling it depends on the amount of u applied at time t. The objective is to minimize the total cost of the invasion:

$$\min_{u_t} E\sum_{t=0}^{\infty} \left[C(u_t) + D(x_t)\right](1+\delta)^{-t} \tag{1}$$

subject to:

$$y_{t+1} = \left[x_t + \Delta y_t(x_t)\right]\rho \tag{2}$$

$$x_t = y_t - u_t \tag{3}$$

$$\Delta y_t(x_t) = r x_t\left(1 - \frac{x_t}{K}\right) \tag{4}$$

$$C(u_t) = \beta_C u_t \tag{5}$$

$$D(x_t) = \beta_D x_t \tag{6}$$

Where y_t is the size of the invasion before control has been applied in time t. The total cost in (1) includes the cost of damage (D) caused by the invasion and the cost of controlling the invasion (C). Optimisation of the objective function (1) is subject to the spread of the invasion (2) and to the effect of control on the spread of the invasion (3). The variable ρ represents a random environmental disturbance ($0<\rho<\infty$), which may cause actual spread to be larger or smaller than its expected value. The spread function is given by equation (2) as $\Delta y_t(x_t)$ and is based on a logistic equation (4) with specific growth rate r and carrying capacity K.

The cost and damage functions were assumed to be linear in (5) and (6), the simplest possible form. If we measure the state of the invasion (x_t) in hectares, then cost and damage coefficients (β_C and β_D) are expressed as $/ha and are independent of the size of the invasion. This assumption will be relaxed in future work.

To solve the problem analytically[1] we set the stochastic shock ρ=1 and substitute (3) and (4) into (2) to obtain:

$$x_{t+1} = r x_t\left(1 - \frac{x_t}{K}\right) - u_{t+1} + x_t \tag{7}$$

So problem (1) becomes:

$$\min_{u_t} E\sum_{t=0}^{\infty} \left[\beta_C u_t + \beta_D x_t\right](1+\delta)^{-t}$$

[1] This work was partly completed during a short visit to the Australian Centre for Biosecurity and Environmental Economics. The contribution of Hoang Long Chu and Tom Kompas to this work is gratefully acknowledged. Any errors are my sole responsibility.

subject to:

$$x_{t+1} = x_t + r\left(1 - \frac{x_t}{K}\right)x_t - u_{t+1}$$

The Hamiltonian function for this problem is:

$$H_t = \left[\left(\beta_C u_t + \beta_D x_t\right) + \lambda_t \left[x_t + r x_t \left(1 - \frac{x_t}{K}\right) - u_{t+1} + x_{t+1} \right] \right] \left(1 + \delta\right)^{-t} \quad (8)$$

The first order conditions (FOC) for minimisation are:

with respect to u_t: $\beta_C - \lambda_{t-1} = 0$ (9)

with respect to x_t: $\beta_D + \lambda_t \left(1 + r - 2r \frac{x_t}{K}\right) - \left(1 + \delta\right)\lambda_{t-1} = 0$ (10)

with respect to λ_t: $x_t - r x_t \left(1 - \frac{x_t}{K}\right) - u_{t+1} - x_{t+1} = 0$ (11)

The Hamiltonian function is not convex so the FOC do not result in and interior minimum. The solution results in bang-bang control, where control switches between corner solutions. This is how the switching point is derived.

From (9) we have $\lambda_t = \beta_C$ for all t, which implies that the marginal cost of removing one extra unit of pest (β_C) is equal to the shadow cost of having one extra unit of pest (λ). The shadow cost captures the future cost of spread caused by allowing the pest to remain in the area for an extra time period. From (10) we have:

$$\beta_D + \beta_C \left(1 + r - 2r \frac{x_t}{K}\right) - \left(1 + \delta\right)\beta_C = 0$$

which can be rearranged to:

$$\beta_D + \beta_C \left(r - 2r \frac{x_t}{K}\right) = \delta\, \beta_C \quad (12)$$

The LHS of (12) is the marginal benefit of removing one extra unit of pest. It consists of the avoided damage (β_D) and the avoided cost of removing the pest in the future. The RHS of (12) is the marginal cost of removing one extra unit of pest; it is the opportunity cost (forgone interest) of the removal expense.

The switching point (also known as a Skiba point in the literature), is given by the intersection of the marginal benefit (MB) and the marginal cost (MC) curves as represented in Figure 1. That intersection corresponds to the equality in (12).

At any given time, if the infestation is at $x_t < x_S$, then MB > MC and the pest should be removed, leading to eradication.

At any given time, if the infestation is at $x_t > x_S$, then MB < MC and the pest should be allowed to spread.

The switching point is derived from equation (12) as:

$$x_S = \frac{\beta_D + (r - \delta)\beta_C}{2\beta_C\, r} K \qquad (13)$$

Some interesting insights can be obtained through further analysis of (13), but this is left for future reports.

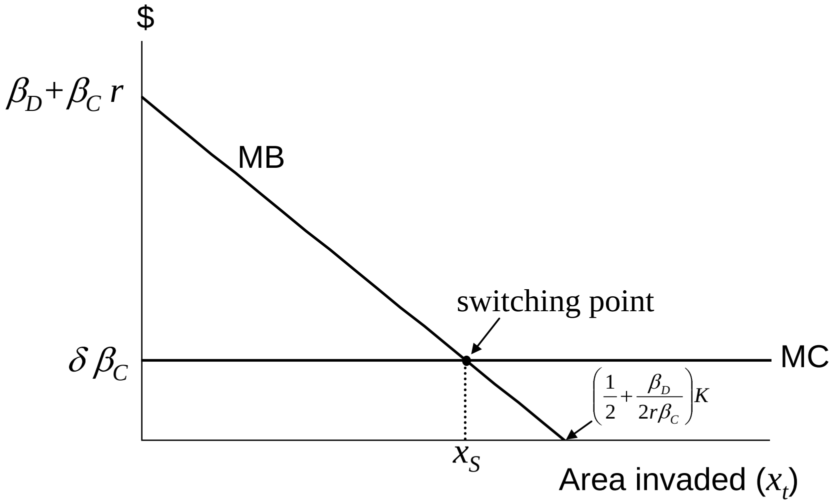


Figure 1.Graphical representation of equation (12) and the location of the switching point.

4. References

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