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Author(s) / Address (es)		
Tracy M. Rout, Yacov Salomon and Michael A. McCarthy		
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Summary		
<p>This study outlines a method for supporting decisions to declare that an eradication program has been successful. Previous approaches to this problem have depended on an estimate of the detectability of the species in a standard survey. This study circumvents this issue by analysing the record of 'absence' results. The problem is solved by minimising the net expected cost of the decision.</p> <p>The study finds the optimal number of surveys after which to declare eradication using three approaches: a stochastic dynamic program, which finds the exact solution, a rule of thumb, and an approximation. Both the rule of thumb and approximation give results that are close to the exact optimal solution. The rule of thumb is less reliable.</p> <p>The approximation is a simple calculation, making it an accessible tool that could be applied routinely by managers of eradication programs for invasive species.</p>		
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Hugh Possingham, University of Queensland

Using sighting records to declare eradication of an invasive species

Tracy M. Rout*, Yacov Salomon and Michael A. McCarthy

Commonwealth Environment Research Facility (Applied Environmental Decision Analysis), School of Botany, University of Melbourne, Parkville, 3010, Australia

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Executive Summary

A major challenge for eradication managers is deciding when a program can be deemed successful. Regan et al. (2006) were the first to pose this problem within a decision theory framework, minimising the net expected cost of the decision. The optimal time to declare eradication was based on the number of consecutive surveys in which the species was not found ('absent surveys'). Their formulation used estimates of detectability and persistence—parameters that are often difficult to estimate—in order to calculate the probability that the invasive species is still present.

Here we use a similar decision-making framework but instead predict presence based on the pattern of sightings using a method developed by Solow (1993a) and a modification in which the pre-extinction sighting rate declines. This method does not require estimates of detectability and persistence. We find the optimal number of absent surveys after which to declare eradication using three approaches: a stochastic dynamic program, which finds the exact optimal solution, a rule of thumb, and an approximation. We compare this with results using a different method for calculating the probability of presence, which assumes the sighting rate declines.

Both the rule of thumb and approximation give results that are close to the exact optimal solution. The rule of thumb with the declining sighting rate generally gives a larger optimal number of absent surveys.

Analysing this problem within a decision theoretic framework enables us to minimise the expected cost of declaring eradication. By using the more readily available sighting data, we make this framework applicable to a wider range of invasive species. Our approximation is a simple calculation, making it an accessible tool that could be applied by managers of eradication programs for invasive species.

Introduction

Eradication requires the removal of every individual of a species from a target area—for plants this entails the removal of both adults and seeds. There have been many successful eradications of both animals and plants (Mack & Lonsdale, 2002; Simberloff, 2002; Simberloff, 2003). A large challenge facing eradication managers is deciding when a program can be declared successful (Morrison et al., 2007). Survey techniques are imperfect, so the failure to detect a species does not necessarily mean it is absent. An invasive species can re-emerge if eradication is declared prematurely, and monitoring stopped, with resulting ecological impacts and costs of further management. Despite this risk, eradication is still declared on an ad hoc basis (Regan et al., 2006), for example after 3 years without detection (Rejmanek & Pitcairn, 2002).

Regan et al. (2006) took an economic approach to the question of when to declare eradication, using decision theory. They found the stopping time (based on the number of previous consecutive surveys in which the species is not found, hereafter referred to as absent surveys) that minimises the net expected cost. This is essentially a trade-off between the cost of continued surveying and the cost if eradication is declared when the species is still present.

Although this work represents a new way of thinking about how we approach setting guidelines for invasive species eradication, its practicality is reduced by the data requirements of the model. To calculate the probability that an invasive species is still present after a number of absent surveys, Regan et al. (2006) used probabilities of persistence and detection. These parameters are difficult to estimate for many invasive species. For example, in the field of population viability analysis, uncertainty around the estimates of probabilities of persistence

can span zero and one (Ludwig, 1999; McCarthy, Burgman & Ferson, 1996). Similarly, methods for estimating detection probabilities usually require labour intensive data (MacKenzie et al., 2002; Tyre et al., 2003; Wintle et al., 2004), and detection probabilities for newly invading species are likely to be very uncertain. Instead of estimating these parameters, we can use the presence-absence sighting record of the species.

There are several methods documented in conservation literature that use a species' sighting record to infer persistence. Solow (1993a) used a presence-absence sighting record, and assumed a constant pre-extinction sighting rate. This essentially assumes the species population level remains constant prior to extinction. Solow (1993b) described a variation of the equation for use in declining populations where the pre-extinction sighting rate declines. Both these methods assumed sightings are independent of one another, and approximately random. Solow & Helser (2000) provided a summary and discussion of these methods. Solow & Roberts (2003) described a nonparametric test, based on the two most recent sightings of the species. All these methods, and variations, are summarised in Solow (2005).

In addition to these, several papers have focused on using sighting records and collection data to identify declining or threatened species. Burgman, Grimson & Ferson (1995) used a discrete form of Solow's equation (Solow, 1993a), extended to account for multiple sightings within one time period. They also explored methods that are sensitive to patterns in sighting data, and use a runs test (Grimson, Aldrich & Wanzer Drane, 1992) to calculate the probability of the longest period for which the species was not observed. McCarthy (1998) used five different methods of identifying declining species from museum records, including Solow's equation, a runs test, a trend analysis, and partial versions of the trend analysis and Solow's equation, which account for variable collection effort. McInerny et al. (2006) commented that Solow's equation may not detect extinction of recently discovered species (those with a short initial sighting period). They modified the equation to remove the influence of the length of this initial sighting period.

Two other studies tested these statistical methods and applied them to real data sets. Burgman et al. (2000) calculated the power of Solow's equation and the runs test by applying them to data generated from a scenario where the 'true' rate of population decline was known. They found both equations had a type 1 error rate (probability of detecting a decline when there is none) of less than or equal to the conventional limit of 0.05. They also applied Solow's equation, the runs test, and the partial Solow equation (McCarthy, 1998) to herbarium data for all Western Australian *Acacia* species. Robbirt, Roberts & Hawkins (2006) used herbarium data for endemic Ecuadorian species of *Guzmania* (Bromeliaceae) to apply five different methods: Solow's equation, the Solow & Roberts equation (Solow & Roberts, 2003), partial versions of Solow's equation and the Solow & Roberts equation, and the sighting rate equation from McInerny et al. (2006). They found a close to significant rank correlation ($p < 0.1$) between the partial equations, which account for collection effort, and IUCN red list categories.

We use the Bayesian formulation of the equation from Solow (1993a) to calculate the probability that a weed is still extant, given a presence-absence sighting record. This equation assumes a constant pre-extinction sighting rate. We incorporate this into the decision-making framework of Regan et al. (2006) to determine the optimal number of absent surveys after which eradication should be declared. We first examine the analytical solution of the new equation, and find a simple approximation for when to declare eradication. We then use stochastic dynamic programming to find an exact optimal solution that incorporates the possibility that the weed may be seen in future surveys. We also include a solution assuming the pre-extinction sighting rate declines. We apply these methods to the example of bitterweed (*Helenium amarum*), the same case study used by Regan et al. (2006).

Methodology

We apply the method from Solow (1993a) to cases in which active surveys for a species are conducted. The sighting record is assumed to follow a Poisson process with a constant but unknown pre-extinction sighting rate m . We use the number of surveys as a unit for the sighting record. A total of S surveys have been performed, and the species has been sighted in n of these (s_1, s_2, \dots, s_n). The species has not been seen for the last $S - s_n$ surveys. These surveys do not occur with reference to time, but they must occur far enough apart as to be independent observations. The probability the species is extant given its sighting record \mathbf{s} is:

$$p(\text{extant} | \mathbf{s}) = (1 + \{(1 - \pi)/[\pi B(\mathbf{s})]\})^{-1}, \quad (1)$$

where π is the prior probability the species is extant, independent of the sighting record (Solow 1993a). $B(\mathbf{s})$ is the Bayes factor, the ratio of likelihoods:

$$B(\mathbf{s}) = l(\mathbf{s} | \text{extant})/l(\mathbf{s} | \text{extinct}),$$

which, is (Solow 1993a):

$$B(\mathbf{s}) = (n - 1)/[(S/s_n)^{n-1} - 1]. \quad (2)$$

Rule of thumb

Regan et al. (2006) outlined a ‘rule of thumb’ for finding when an invasive species should be declared eradicated. Their framework is based on finding the number of consecutive surveys in which the species is not found after which eradication should be declared. The optimal number of consecutive absent surveys is that which gives the lowest net expected cost (NEC). The NEC of stopping after an absent survey is the cost of surveying, plus the expected cost of escape and damage if the species was present but went undetected (Regan et al., 2006). The net expected cost (NEC) of stopping after d absent surveys (where $d = S - s_n$) is:

$$NEC(d) = (d - 1)C_s + p(d)C_e,$$

where C_s is the cost of one survey, C_e is the expected cost of escape and damage, $p(d)$ is the probability that the species is present after d absent surveys.

In Regan et al. (2006), $p(d)$ is given by $[h(1 - q)]^d$, where h is the annual probability the species persists and q is the annual probability of detection. Instead, we substituted $p(d)$ with equation 1 to give:

$$NEC(d) = (d - 1)C_s + C_e/(1 + \{(1 - \pi)/[\pi B(\mathbf{s})]\}). \quad (3)$$

where $B(\mathbf{s})$ is given by equation 2. The minimum NEC (and thus the optimal value of d , denoted d^*) occurs where the derivative of this equation with respect to d is equal to 0, that is:

$$C_s + \frac{C_e (n - 1)^2 (\pi - 1) \pi y^n}{s_n ((1 - \pi) y^n - (1 - n \pi) y)^2} = 0, \quad (4)$$

where $y = (s_n + d^*)/s_n$. We cannot rearrange this equation to give d^* directly, so instead we find d^* numerically by calculating equation 3 for a range of integer values of d and identifying the value with the lowest NEC.

Approximation

We can also derive an approximate direct expression for d^* . We cannot rearrange equation 4 for d^* , but we can rearrange it to find the value of the cost ratio R (i.e. $R = C_s/C_e$) where the NEC is minimised. We name this value R_{crit} , a critical value of R at which it becomes optimal to stop surveying, i.e. for $R < R_{crit}$ we should keep surveying, while for $R > R_{crit}$ we should stop. R_{crit} is:

$$R_{crit} = \frac{(n-1)^2(1-\pi)\pi y^n}{s_n \left[(1-\pi)y^n - (1-n\pi)y \right]^2},$$

and its natural log is:

$$\ln(R_{crit}) = 2\ln(n-1) + \ln(1-\pi) + \ln(\pi) + n\ln(y) - 2\ln((1-\pi)y^n - (1-n\pi)y).$$

Where y is large, $\ln(R_{crit})$ is approximately linear with respect to $\ln(y)$. A Taylor series approximation of $\ln(R_{crit})$ around a large value of y gives:

$$\ln(R_{crit}) \approx 2\ln(n-1) + \ln(\pi) - \ln(1-\pi) - n\ln(y),$$

which means

$$R_{crit} \approx \frac{(n-1)^2\pi}{s_n(1-\pi)y^n}.$$

We can then substitute $y = (s_n + d^*)/s_n$ and rearrange to get the expression:

$$d^* \approx s_n \left(\frac{(n-1)^2\pi}{s_n(1-\pi)R} \right)^{\frac{1}{n}} - s_n. \quad (5)$$

This approximation to the rule of thumb can be used to find the approximate optimal value of d (not necessarily an integer value), without repeatedly calculating the net expected cost.

Stochastic dynamic programming

The rule of thumb (equation 3) and its approximation (equation 5) do not include the possibility that the species may be seen in a future survey, incurring further costs of surveying and possible escape and damage. To incorporate these future expected costs, we can use stochastic dynamic programming. Stochastic dynamic programming is an optimisation algorithm that can be applied to any system with a finite number of states, where the dynamics are described by a Markov chain and sequential decisions must be made (Bellman, 1957; Lubow, 1996; Mangel & Clark, 1988). It works backwards over time, finding optimal decisions for each possible management scenario that take into account future expected costs (Bellman, 1957; Lubow, 1996; Mangel & Clark, 1988).

The formulation of our stochastic dynamic program (SDP) is similar to that in Regan et al. (2006). In each time step m (1 to M) there are two possible management decisions: to survey or to stop. The optimal decision is the one with the lowest expected cost. As outlined previously, the species has a sighting record in which it is seen n times in s_n surveys, and then not seen for d surveys. The optimal stopping time for particular values of n and s_n is the smallest d where the expected cost of stopping is less than the expected cost of surveying. The expected cost of stopping is the probability that the species is extant given its sighting record, multiplied by the expected cost of escape and damage:

$$E_{stop}(m, d, n, s_n) = p(\text{species extant} | n, s_n, d)C_e,$$

which substituting equation 1 becomes:

$$E_{stop}(m, d, n, s_n) = C_e / (1 + \{(1 - \pi) / [\pi B(\mathbf{s})]\}),$$

where $B(\mathbf{s})$ is given by equation 2. The expected cost of surveying must encompass two possibilities: the species is detected or not detected. The sighting record can be updated for each case. If the species is detected, the number of sightings n becomes $n + 1$, while the most recent sighting s_n becomes $s_n + d$. The number of absent surveys d becomes 0. If the species is not detected, d becomes $d + 1$, while n and s_n remain constant. The expected cost of surveying is thus:

$$\begin{aligned} E_{survey}(m, d, n, s_n) &= C_s + p(\text{extant})p(\text{detected})E_{opt}(m + 1, 0, n + 1, s_n + d) + \\ &\quad (1 - p(\text{extant})p(\text{detected}))E_{opt}(m + 1, d + 1, n, s_n) \quad (m < M) \\ &= C_s \quad (m = M). \end{aligned}$$

The probability that the species is extant is given by equation 1. E_{opt} is the expected cost of future optimal decisions, where the optimal decision gives the lowest expected cost:

$$E_{opt}(m, d, n, s_n) = \min[E_{stop}(m, d, n, s_n), E_{survey}(m, d, n, s_n)].$$

If the species went extinct immediately after it was last detected in survey s_n , its probability of detection would be n/s_n . However, if the species was extant but undetected for k of the d absent surveys, the detectability estimate would be $n/(s_n + k)$. The value of k could be anywhere between 0 and d . We can determine a general probability of detection by weighting the estimates for each possible k by the probability that the species was present at survey $s_n + k$, but then went extinct before $s_n + k + 1$:

$$p(\text{detected} | n, s_n, d) = \sum_{k=0}^d \left[\left(\frac{n}{s_n + k} \right) (p(\text{extant} | n, s_n, k) - p(\text{extant} | n, s_n, k + 1)) \right].$$

The SDP is calculated backwards, so it begins by calculating the expected cost of each decision in the final time step M . The optimal decision is the one with the lowest expected cost. It then steps back to the previous time step $M - 1$ to calculate the expected cost of each decision, assuming the optimal action is taken in time step M . It continues to step backwards, repeating this process and finding the optimal decision for each time step. The optimal decision for the first time step thus accounts for future possibilities and costs, assuming that all future decisions are optimal. For this reason, we have focused our analysis of SDP results on those from the first time step.

Alternative sighting rate model

The methods described above assume the sighting record of the species represents a Poisson process, with a constant pre-extinction sighting rate. A plausible alternative model is that the pre-extinction sighting rate λ is not constant, but declines with the number of surveys s . We define $\lambda(s)$ as:

$$\lambda(s) = ms^{-a}, \quad (6)$$

where m is a constant, and a is a constant between 0 and 1 (where $a = 0$ corresponds to the constant rate case above). We chose a simpler sighting rate function than that investigated by Solow (1993b), to enable us to calculate the Bayes factor. The Bayes factor for this model (integrated over all possible values of m) is:

$$B(\mathbf{s}) = (n(1-a) - 1) / [(S/s_n)^{n(1-a)-1} - 1].$$

Details of this calculation are provided in Appendix A. To investigate the optimal decisions under this model, we substituted this Bayes factor into the rule of thumb (equation 3) to calculate the net expected cost of stopping after d absent surveys.

Case study

We apply the method to the eradication of *Helenium amarum* (bitterweed) in Queensland, Australia—the example that was used in Regan et al. (2006). *H. amarum* is toxic to stock, and if ingested causes vomiting, diarrhoea, and production of bitter undrinkable milk. It was first found in Queensland in 1953, and an eradication program began in the same year. After three years of herbicide and manual removal, only isolated patches of plants remained. Between 1988 and 1992 no plants were detected, and the weed was declared eradicated (Tomley & Panetta, 2002). This eradication seemed to have been successful (Regan et al. 2006) but in March 2007 a small infestation of *H. amarum* was discovered at the site of original occupancy, and control activities are currently underway (D. Panetta pers. comm.).

We used the best estimate parameters in Regan et al. (2006) and the raw sighting data of *H. amarum* to parameterise the models. The best estimate for the cost ratio R (C_s/C_e) is 0.00282, meaning the cost of escape is 354 times the cost of surveying (Regan et al. 2006). From 1953 until the last sighting in 1987, 169 surveys were conducted ($s_n = 169$), and *H. amarum* was seen in 142 of those surveys ($n = 142$). *H. amarum* was declared eradicated after 9 consecutive absent surveys in the period 1988-1992 ($d = 9$). We used a non-informative prior probability of presence \square of 0.5. In the SDP we found optimal decisions for twenty consecutive choices ($M = 20$), and we discuss the optimal decisions for the first of these as they incorporate the possible future outcomes of the next 19 choices. For the alternative model of sighting rate, we used WinBUGS software (Spiegelhalter et al., 2003) to fit the declining sighting rate model (equation 6) to the raw sighting data for *H. amarum*. The estimate was $a \approx 0.1$ when using a uniform prior.

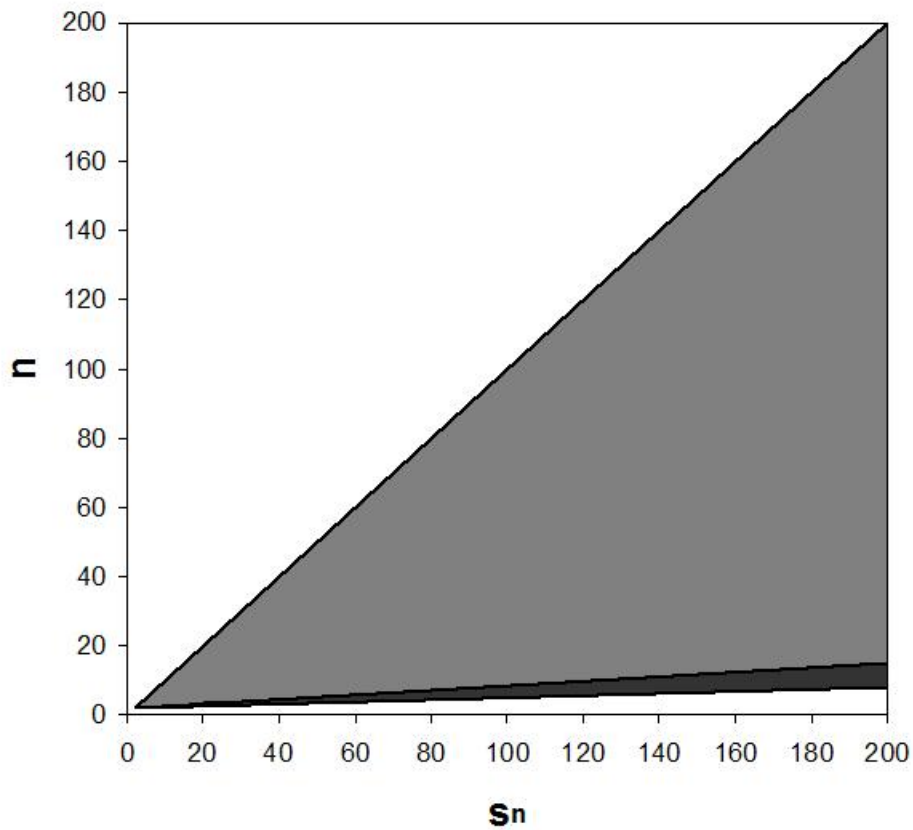
Results

Performance of rule of thumb and approximation

The exact optimal number of absent surveys before stopping (calculated with the SDP) decreases as the sighting frequency n/s_n increases (Figure S1). For most combinations of n and s_n , the rule of thumb gives results that are within one absent survey of the exact optimal results from the SDP (Figure 1a). Larger differences occur when the sighting frequency is very

low, with the rule of thumb underestimating the exact optimal result. Similarly, the approximation gives results that are within one absent survey of the exact optimal for most combinations of n and s_n , and tends to underestimate the optimal result when the sighting frequency is very low (Figure 1b). The approximation also tends to slightly overestimate the optimal result (by up to 3 surveys) when the sighting frequency is moderately low.

a)



b)

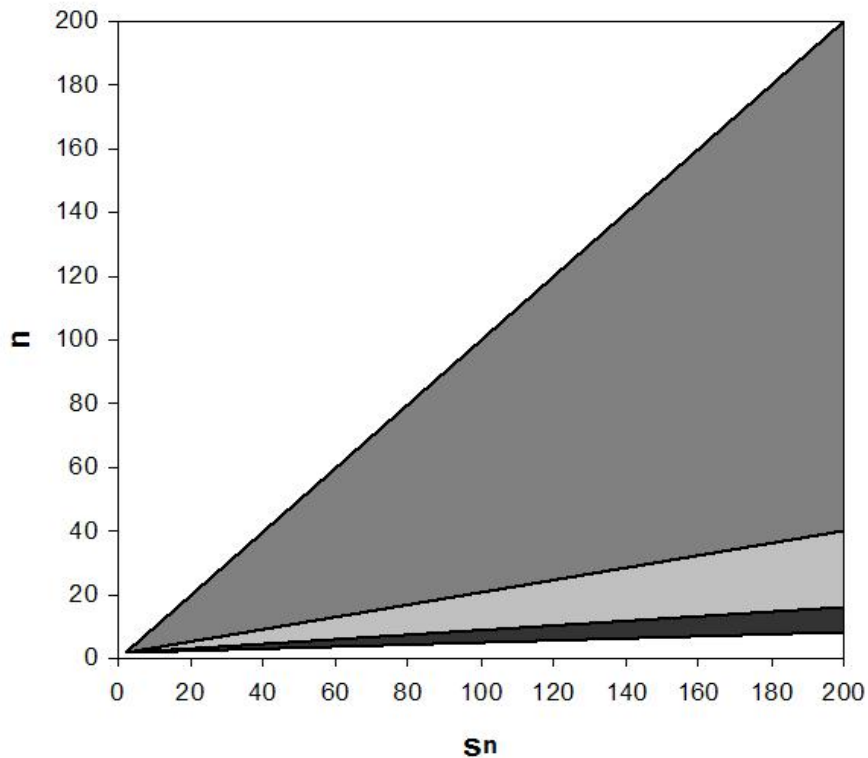


Figure 1: Performance of the a) the rule of thumb and b) the approximation as compared to the exact optimal results from the SDP. Mid grey shading indicates combinations of n and s_n where the result was within one absent survey of the SDP result. Dark grey shading indicates where the optimal result was underestimated by more than one absent survey (up to 134 surveys for both methods), and light grey shading indicates where the optimal result was overestimated (by up to 2.23 surveys).

Using the constant sighting rate model, the rule of thumb, SDP and approximation all give an optimal stopping time of 13 absent surveys ($d^* = 13$) for *H. amarum* (Figure 2: rule of thumb, Figure S2: SDP, approximation $d^* = 13.15$). The SDP finds the optimal decision, to keep surveying or stop, for every possible sighting record (every combination of n , s_n and d). If there have been 12 consecutive absent surveys, it is optimal to stop surveying for many combinations of n and s_n , but not for the initial sighting record of *H. amarum* (Figure S2a). After 13 consecutive absent surveys, it is optimal to stop surveying for *H. amarum* (Figure S2b). As the number of absent surveys increases, the number of combinations for which it is optimal to stop surveying increases.

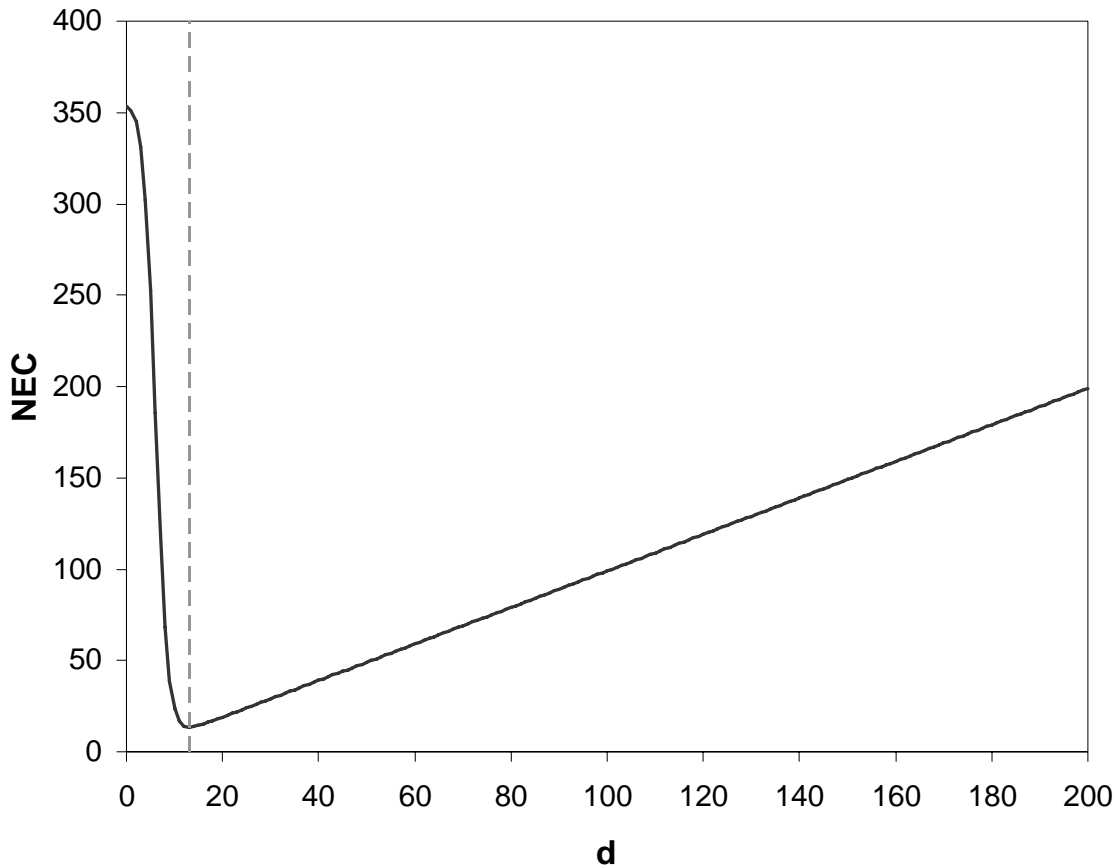


Figure 2: The net expected cost (NEC) as a function of the number of absent surveys for *H. amarum*, calculated using the rule of thumb (equation 3). The lowest NEC occurs after 13 years of absent surveys (marked with dotted line), making this the optimal number after which to declare eradication.

Sensitivity of optimal solution

The results from the SDP are sensitive to the ratio of the cost of surveying to the cost of escape ($R = C_s/C_e$). For a particular combination of n and s_n , decreasing this ratio R increases the optimal number of absent surveys. For $s_n = 200$, the results for $R = 1$ and $R = 0.1$ are the same—stop surveying even if the weed has been seen in the previous survey ($d^* = 0$) (Figure 3). For smaller cost ratios ($R = 0.01, 0.001, 0.0001$), the optimal number of absent surveys decreases as n increases (Figure 3). The difference between results for each cost ratio is greater for smaller values of n (Figure 3).

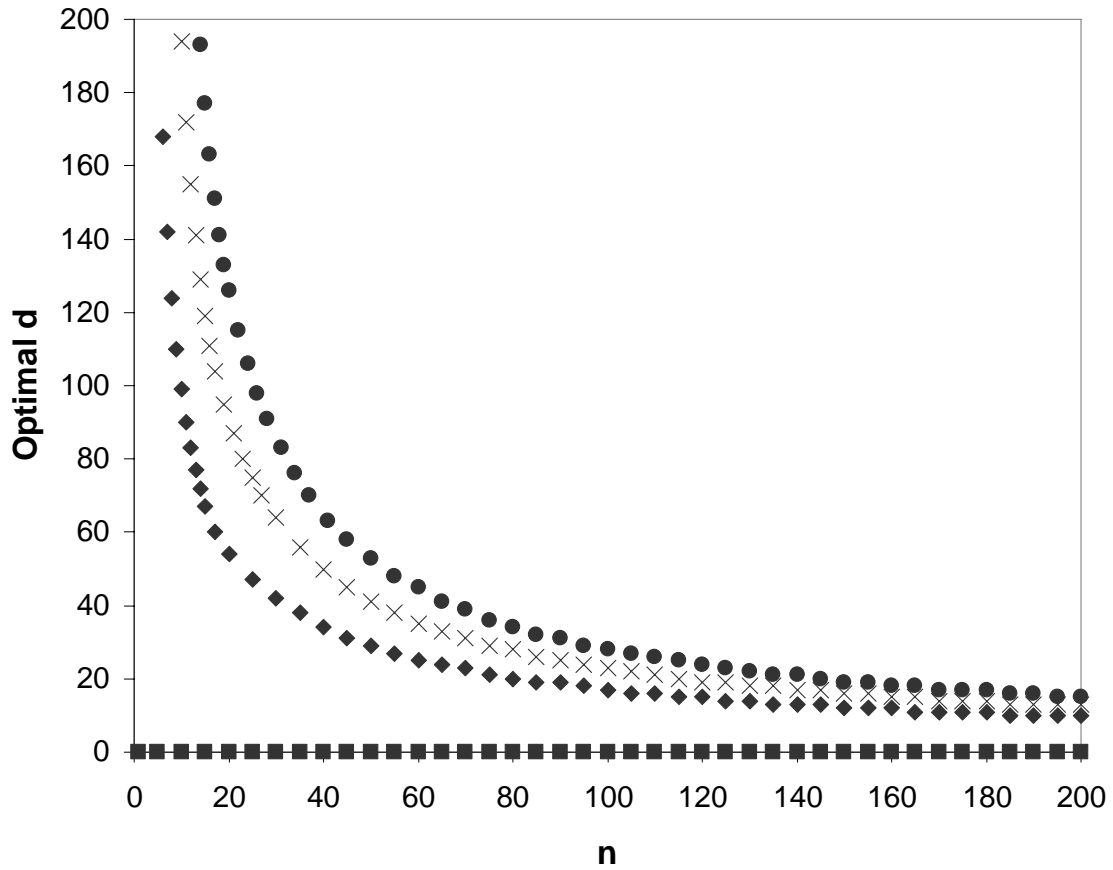


Figure 3: SDP results for different values of the cost ratio R (C_s/C_e), for $s_n = 200$. Squares: $R = 1$ or 0.1 , diamonds: $R = 0.01$, crosses: $R = 0.001$, circles: $R = 0.0001$. The prior probability of presence is kept constant at $\pi = 0.5$.

The SDP results are also sensitive to the prior probability of species presence (π). Higher values of the prior give a larger optimal number of absent surveys (Figure 4). As with the cost ratio, the difference is larger for small values of n . The difference is also greater around the edges of the range for π , that is, when π is close to 0 or 1. For example, the difference in results between $\pi = 0.9$ and $\pi = 0.9999$ is similar to that between $\pi = 0.1$ and $\pi = 0.9$.

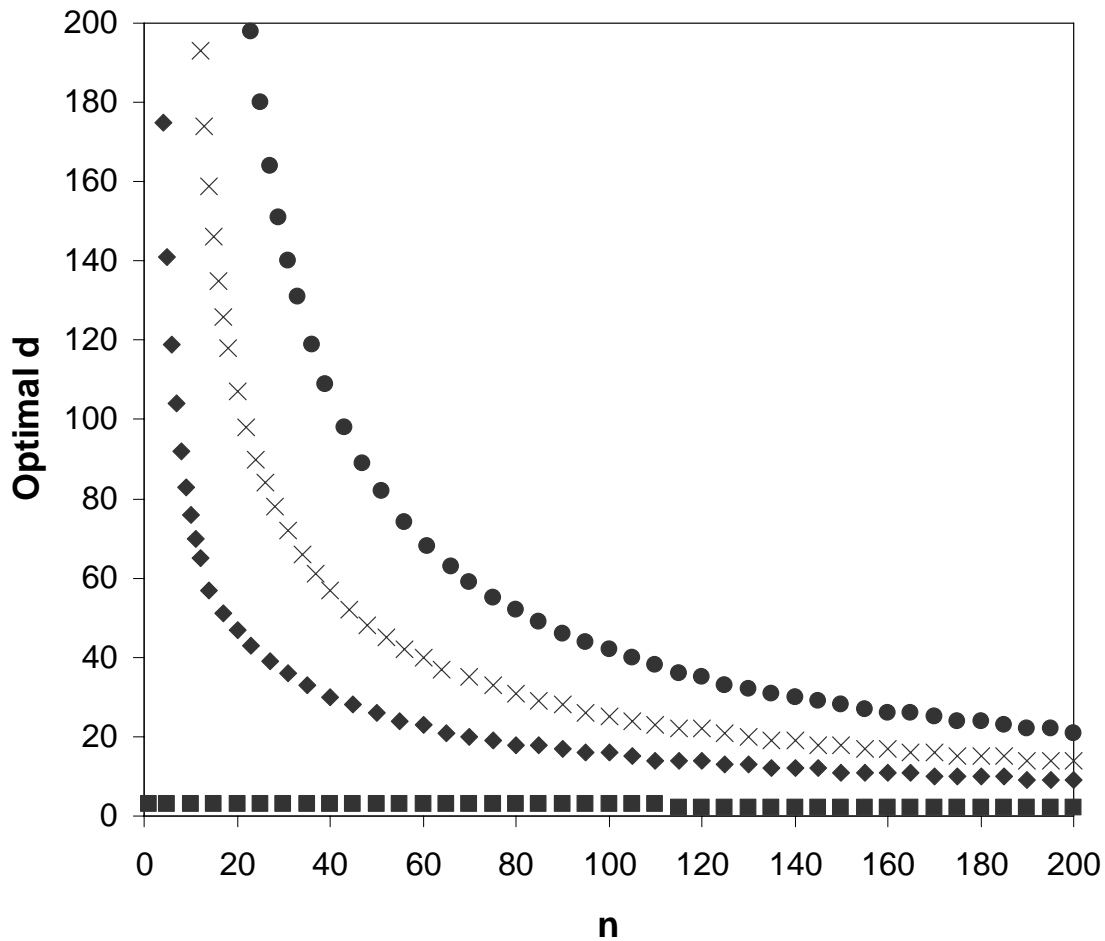


Figure 4: SDP results for different values of the prior π , for $s_n = 200$. Squares: $\pi = 0.0001$, diamonds: $\pi = 0.1$, crosses: $\pi = 0.9$, circles: $\pi = 0.9999$. The cost ratio is kept constant at $R = 0.00282$.

Declining sighting rate model

When using the parameters for *H. amarum* (including $a = 0.1$) the rule of thumb with the declining sighting rate model gave an optimal d of 14, as compared with 13 from the original model. For any value of d , the declining sighting rate equation gives a higher probability that the species is extant than Solow's equation, although both give much higher probabilities than the methods used in Regan et al. (2006) (Figure 5).

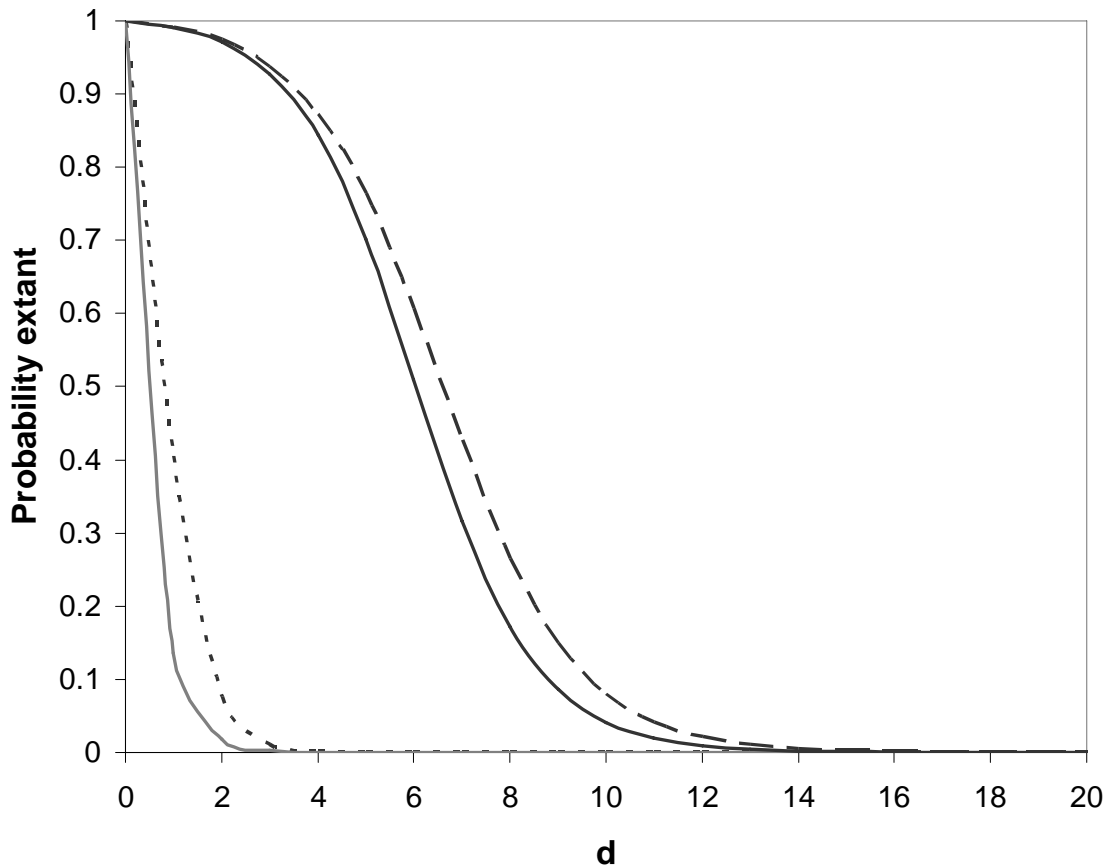


Figure 5: The probability that *H. amarum* is extant as a function of the number of consecutive absent surveys (shown up to $d = 20$), calculated with four different methods. The solid grey line is calculated as in the rule of thumb in Regan et al. (2006), and the dotted line is calculated as in the SDP in Regan et al. (2006). Both of these used the best estimate parameters for *H. amarum* described in Regan et al. (2006), and assumed surveys are conducted annually. The solid black line is calculated using Solow's equation, and the dashed line is calculated using the declining sighting rate equation. Both of these used the sighting record for *H. amarum* ($s_n = 169$, $n = 142$), and $\pi = 0.5$ as the prior probability it is extant. The declining sighting rate equation used $a = 0.1$.

The parameter a in this model represents the magnitude of decline in sighting rate over the original sighting period. When $a = 0$ this model is equivalent to the constant sighting rate model, and it gives the same optimal number of absent surveys (Figure 6). The optimal number of absent surveys then increases as a increases, but the increase up to $a = 0.5$ is gradual: when $a = 0.5$, $d^* = 23$.

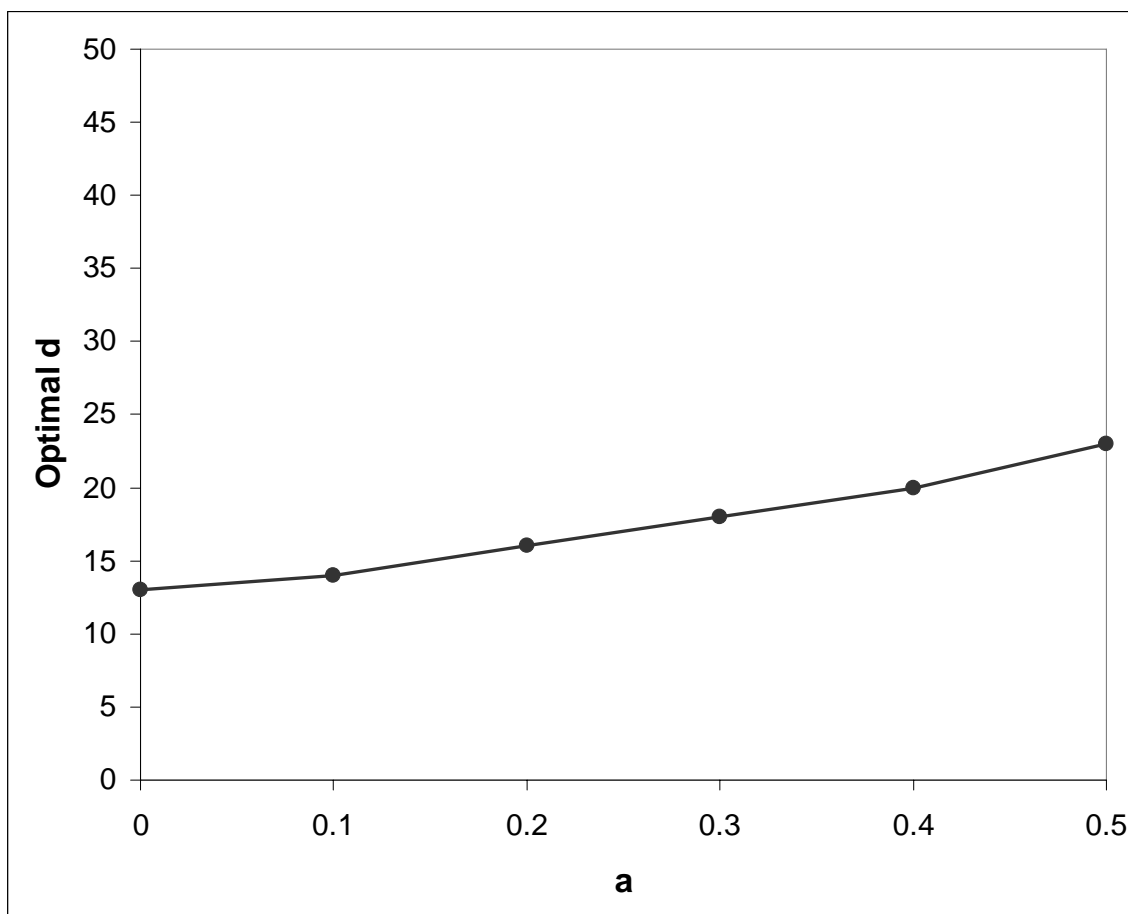


Figure 6: The optimal number of consecutive absent surveys found using the declining sighting rate equation, for different values of parameter a . All other parameters values are set for *H. amarum*. When $a = 0$, the declining sighting rate equation is equivalent to Solow's equation.

Discussion

Regan et al. (2006) provided the first decision theoretic method for determining when to declare eradication of an invasive species. We examine alternative methods for calculating the probability an invasive species is extant, for use in their decision making framework. Although our results are not directly comparable with those in Regan et al. (2006) as they are in different units (surveys vs. annual surveys), it is clear that the method used to calculate this probability influences the optimal decision of when to declare eradication. The question is then: which method is more suitable? A benefit of methods that use sighting data is that they are amenable to power analysis through simulation, so their accuracy in different circumstances can be determined (see Burgman et al. 2000). Otherwise, the method used may simply depend on the type of data available. Regan et al.'s (2006) method requires estimates of annual detectability and annual probability of persistence. When these estimates are not available, methods that make use of the raw sighting data may be preferable. However, they instead require an estimate of the prior probability of presence.

The prior probability of presence could be obtained by eliciting probabilities from one or more experts (McCarthy, 2007), or by summarising the success and failure of similar eradication attempts. For example, in a review of goat eradications from islands, Campbell & Donlan (2005) found 120 documented successes and 10 documented failures. In this case the prior probability of an unsuccessful eradication (and thus the prior probability that goats are still

present on an island following an eradication attempt) is 0.08. In some cases, probability of eradication can be predicted, a priori, as a function of characteristics of the site or species. For example, Campbell & Donlan (2005) show that eradication of goats from larger islands was less successful than from smaller islands. Similarly, eradication success will be a function of the level of effort. McCarthy & Masters (2005) illustrate this approach for predicting annual survival rates of birds by constructing regression models based on previously published data. A meta-analysis of the eradication literature would provide a means for generating prior probabilities of eradication failure, based on the characteristics of the eradication program.

In the absence of such a meta-analysis, we initially chose a prior of $\pi = 0.5$ for *H. amarum*, and analysed the results over the entire range of π . We found the exact optimal results were sensitive to this prior probability, with sensitivities being more pronounced where the initial sighting frequency was very low. The exact optimal results were also more sensitive to changes in the cost ratio R when the sighting frequency was very low. For cryptic species with fewer sightings in the original sighting period, it is important to have reliable estimates of these parameters.

We have explored here three different methods for finding the optimal number of absent surveys before declaring eradication: the SDP, rule of thumb, and an approximation. These three methods trade-off accuracy and accessibility. The SDP calculates the exact optimal result, but the method and the results it produces are both quite difficult to understand and interpret. Computer programming skills are required to conduct the analysis and summarise the results. The rule of thumb is more intuitive, but still requires time and some technical skills to calculate the net expected cost over a range of values of d to find the minimum. The approximation is a simple calculation that could be performed quickly with a calculator, making it an accessible tool that could be applied by decision-makers and managers of weed eradication programs. The rule of thumb and approximation perform well when compared with the exact optimal results from the SDP. Accuracy of these methods is only a concern for the most cryptic species where the initial sighting frequency is very low.

As mentioned in the methods section, we have applied Solow's equation to active surveys of species. In Solow (1993a), the pre-extinction sighting rate is constant over time, which is not sensible when surveys are being carried out—it implies the probability of sighting during a survey is the same as the probability of sighting between surveys. We have instead applied Solow's method using the number of surveys as a unit for the sighting record. This means that the sighting rate is constant for each survey, regardless of how much time has passed in between surveys. However, enough time must pass for each survey to be considered an independent observation. If a species has been surveyed at regular time intervals throughout its sighting period then the number of surveys can be used as an index for time.

The interpretation of the sighting record in units of surveys rather than time has little impact when assuming a constant sighting rate. However, care needs to be taken when interpreting results for our declining sighting rate model. The declining sighting rate can be interpreted as modelling a declining population: as the size of the population decreases the probability of sighting an individual declines. However, the sighting rate does not decline over time, but declines with the number of surveys s . This might be reasonable if eradication efforts respond to the surveys.

We have provided two different models for the pre-extinction sighting rate: constant and declining. The constant sighting rate model is suitable for very small populations that are relatively stable but subject to rapid extinction (Solow, 1993b). This is a reasonable description of a population where declaring eradication is a possibility. We are focusing on populations that have already been reduced to a very low level, so if they are still declining, they are not likely to be declining at a great rate for any length of time. This was apparent when fitting the declining sighting rate model to our case study of *H. amarum*. If the decline in sighting rate is only slight,

our results suggest that the optimal number of absent surveys is not greatly different under the constant sighting rate model. In general, assuming a declining sighting rate increases the optimal number of absent surveys compared with a constant sighting rate.

There are numerous other methods to calculate the probability of presence from sighting data (see Introduction), each with their own assumptions about the sighting record and species population. Within this decision-making framework, we could substitute a number of different methods, depending on the species in question and the type of data available. We are currently examining the range of results that might be achieved given plausible alternative models for how the probability of presence declines with the number of consecutive surveys in which a species is not found. Another way to account for uncertainty as to the most appropriate method of calculating the probability of presence is to find a decision that will deliver a satisfactory outcome and is most robust to uncertainty (Ben-Haim, 2006).

References

- Bellman, R.E. (1957) *Dynamic Programming* Princeton University Press, Princeton, NJ, USA.
- Ben-Haim, Y. (2006) *Information-gap Decision Theory: Decisions Under Severe Uncertainty* Academic Press, New York.
- Burgman, M.A., Grimson, R.C. & Ferson, S. (1995) Inferring threat from scientific collections. *Conservation Biology*, **9**(4), 923-28.
- Burgman, M.A., Maslin, B.R., Andrewartha, D., Keatley, M.R., Boek, C. & McCarthy, M.A. (2000). Inferring threat from scientific collections: power tests and an application to Western Australian *Acacia* species. In *Quantitative Methods for Conservation Biology* (eds S. Ferson & M.A. Burgman), pp. 7-26. Springer-Verlag, New York.
- Campbell, K. & Donlan, C.J. (2005) Feral goat eradications on islands. *Conservation Biology*, **19**(5), 1362-74.
- Grimson, R.C., Aldrich, T.E. & Wanzer Drane, J. (1992) Clustering in sparse data and an analysis of Rhabdomyosarcoma incidence. *Statistics in Medicine*, **11**, 761-68.
- Lubow, B.C. (1996) Optimal translocation strategies for enhancing stochastic metapopulation viability. *Ecological Applications*, **6**(4), 1268-80.
- Ludwig, D. (1999) Is it meaningful to estimate a probability of extinction? *Ecology*, **80**(1), 298-310.
- Mack, R.N. & Lonsdale, W.M. (2002). Eradicating invasive plants: hard-won lessons for islands. In *Turning the tide: the eradication of invasive species* (eds C.R. Veitch & M.N. Clout), pp. 164-72. IUCN SSC Invasive Species Specialist Group, IUCN, Gland, Switzerland and Cambridge, UK.
- MacKenzie, D.I., Nichols, J.D., Lachman, G.B., Droege, S., Royle, J.A. & Langtimm, C.A. (2002) Estimating site occupancy rates when detection probabilities are less than one. *Ecology*, **83**(8), 2248-55.
- Mangel, M. & Clark, C.W. (1988) *Dynamic Modeling in Behavioral Ecology* Princeton University Press, Princeton, NJ, USA.
- McCarthy, M.A. (1998) Identifying declining and threatened species with museum data. *Biological Conservation*, **83**(1), 9-17.
- McCarthy, M.A. (2007) *Bayesian Methods for Ecology* Cambridge University Press, Cambridge, UK.
- McCarthy, M.A., Burgman, M.A. & Ferson, S. (1996) Logistic sensitivity and bounds for extinction risks. *Ecological Modelling*, **86**, 297-303.
- McCarthy, M.A. & Masters, P. (2005) Profiting from prior information in Bayesian analyses of ecological data. *Journal of Applied Ecology*, **42**, 1012-19.
- McInerny, G.J., Roberts, D.L., Davy, A.J. & Cribb, P.J. (2006) Significance of sighting rate in inferring extinction and threat. *Conservation Biology*, **20**(2), 562-67.
- Morrison, S.A., Macdonald, N., Walker, K., Lozier, L. & Shaw, M.R. (2007) Facing the dilemma at eradication's end: uncertainty of absence and the Lazarus effect. *Frontiers in Ecology and the Environment*, **5**(5), 271-76.
- Regan, T.J., McCarthy, M.A., Baxter, P.W.J., Panetta, F.D. & Possingham, H.P. (2006) Optimal eradication: when to stop looking for an invasive plant. *Ecology Letters*, **9**, 759-66.
- Rejmanek, M. & Pitcairn, M.J. (2002). When is eradication of exotic pest plants a realistic goal? In *Turning the Tide: The Eradication of Island Invasives* (eds C.R. Veitch & M.N. Clout), pp. 249-53. IUCN SSC Invasive Species Specialist Group, Gland, Switzerland and Cambridge, UK.
- Robbirt, K.M., Roberts, D.L. & Hawkins, J.A. (2006) Comparing IUCN and probabilistic assessments of threat: do IUCN red list criteria conflate rarity and threat? *Biodiversity and Conservation*, **15**, 1903-12.
- Simberloff, D. (2002). Today Tiritiri Matangi, tomorrow the world! Are we aiming too low in invasives control? In *Turning the Tide: The Eradication of Invasive Species* (eds C.R. Veitch & M.N. Clout), pp. 4-13. Invasive Species Specialist Group of the World Conservation Union (IUCN), Auckland.
- Simberloff, D. (2003) Eradication - preventing invasions at the outset. *Weed Science*, **51**, 247-53.
- Solow, A.R. (1993a) Inferring extinction from sighting data. *Ecology*, **74**(3), 962-64.
- Solow, A.R. (1993b) Inferring extinction in a declining population. *Journal of Mathematical Biology*, **32**, 79-82.
- Solow, A.R. (2005) Inferring extinction from a sighting record. *Mathematical Biosciences*, **195**, 47-55.
- Solow, A.R. & Helser, T. (2000). Detecting extinction in sighting data. In *Quantitative Methods for Conservation Biology* (eds S. Ferson & M.A. Burgman), pp. 1-6. Springer-Verlag, New York.
- Solow, A.R. & Roberts, D.L. (2003) A nonparametric test for extinction based on a sighting record. *Ecology*, **84**, 1329.

- Spiegelhalter, D.J., Thomas, A., Best, N. & Lunn, D. (2003). WinBUGS. In. MRC Biostatistics Unit, Institute of Public Health, Cambridge.
- Tomley, A.J. & Panetta, F.D. (2002). Eradication of the exotic weeds *Helenium amarum* (Rafin) H. L. and *Eupatorium serotinum* Michx. from south-eastern Queensland. In *Proceedings of the 13th Australian Weeds Conference* (eds H.S. Jacob, J. Dodd & J.H. Moore), pp. 293-96. Plant Protection Society of Western Australia, Perth, WA.
- Tyre, A.J., Tenhumberg, B., Field, S.A., Niejalke, D., Parris, K. & Possingham, H. (2003) Improving precision and reducing bias in biological surveys: estimating false-negative error rates. *Ecological Applications*, **13**(6), 1790-801.
- Wintle, B.A., McCarthy, M.A., Parris, K.M. & Burgman, M.A. (2004) Precision and bias of methods for estimating point survey detection probabilities. *Ecological Applications*, **14**(3), 703-12.

Appendix A. Derivation of Bayes factor for declining sighting rate model

A total of S surveys have been performed, and the species has been sighted in n of these ($\mathbf{s} = s_1, s_2, \dots, s_n$). We assume this sighting record follows a non-homogenous Poisson process with rate function $\lambda(s)$ (Cox & Lewis, 1966). The sighting rate declines with the number of surveys s such that:

$$\lambda(s) = ms^{-a},$$

where m is a constant, and a is a constant between 0 and 1.

We define:

$$\begin{aligned} \lambda_{s_1, s_2} &= \int_{s_1}^{s_2} \lambda(s) ds \\ &= \int_{s_1}^{s_2} ms^{-a} ds, \end{aligned}$$

which solves to

$$\lambda_{s_1, s_2} = \frac{m}{1-a} (s_2^{1-a} - s_1^{1-a}).$$

The Bayes factor is the ratio of likelihoods:

$$B(\mathbf{s}) = l(\mathbf{s} | \text{extant}) / l(\mathbf{s} | \text{extinct}).$$

The likelihood of the sighting record given the species is extant is:

$$l(\mathbf{s} | \text{extant}) = \int_0^{\infty} l(\mathbf{s} | m) dP(m),$$

where $P(m)$ is the prior distribution for m . We assume m has a uniform prior distribution (Solow, 1993), such that:

$$dP(m) = \frac{1}{m} dm.$$

For a non-homogenous Poisson process, the likelihood is then:

$$l(\mathbf{s} | \text{extant}) = \int_0^\infty \prod_{i=1}^n \lambda(s_i) e^{-\lambda_{0,s}} \frac{1}{m} dm,$$

which, substituting the expressions for $\lambda(s_i)$ and $\lambda_{0,s}$ becomes:

$$\begin{aligned} l(\mathbf{s} | \text{extant}) &= \int_0^\infty \prod_{i=1}^n m s_i^{-a} e^{-\frac{m s_i^{1-a}}{1-a}} \frac{1}{m} dm \\ &= \prod_{i=1}^n s_i^{-a} \int_0^\infty m^{n-1} e^{-\frac{m s_i^{1-a}}{1-a}} dm. \end{aligned}$$

Integrating this by parts gives:

$$l(\mathbf{s} | \text{extant}) = \prod_{i=1}^n s_i^{-a} (n-1)! \left(\frac{s_i^{1-a}}{1-a} \right)^{-n}$$

The likelihood of the data given that the species is extinct is:

$$l(\mathbf{s} | \text{extinct}) = \int_{s_n}^S l(\mathbf{s} | S_e = s_e) dP(s_e),$$

where s_e is the survey at which the species went extinct (between s_n and S). S_e is uniform random variable of which s_e is a realisation. $P(s_e)$ is the prior distribution for s_e . We assume that, without knowledge of the sighting data, extinction is equally likely to occur during any survey in the sighting period (Solow, 1993):

$$dP(s_e) = \frac{1}{S} ds_e.$$

The function $l(\mathbf{s} | S_e = s_e)$ is the likelihood of the sighting record given $S_e = s_e$, which is:

$$l(\mathbf{s} | S_e = s_e) = \int_0^\infty \prod_{i=1}^n \lambda(s_i) e^{-\lambda_{0,s_e}} dP(m).$$

Substituting this into the main likelihood function gives:

$$l(\mathbf{s} | \text{extinct}) = \int_{s_n}^S \int_0^\infty \prod_{i=1}^n \lambda(s_i) e^{-\lambda_{0,s_e}} dP(m) dP(s_e).$$

If we then substitute the expressions for $\lambda(s_i)$, λ_{0,s_e} , and the prior distributions for s_e and m , this becomes:

$$\begin{aligned}
 l(\mathbf{s} \mid \text{extinct}) &= \int_{s_n}^S \int_0^\infty \prod_{i=1}^n m s_i^{-a} e^{\frac{-m s_e^{1-a}}{1-a}} \frac{1}{mS} dm ds_e \\
 &= \int_{s_n}^S \prod_{i=1}^n s_i^{-a} \frac{1}{S} \int_0^\infty m^{n-1} e^{\frac{-m s_e^{1-a}}{1-a}} dm ds_e \\
 &= \prod_{i=1}^n s_i^{-a} \frac{1}{S} (n-1)! \int_{s_n}^S \left(\frac{s_e^{1-a}}{1-a} \right)^{-n} ds_e \\
 &= \prod_{i=1}^n s_i^{-a} \frac{1}{S} (n-1)! \frac{(1-a)^n}{na-a+1} (S^{na-n+1} - s_n^{na-a+1})
 \end{aligned}$$

The Bayes factor is then:

$$B(\mathbf{s}) = \frac{\prod_{i=1}^n s_i^{-a} (n-1)! \left(\frac{s^{1-a}}{1-a} \right)^{-n}}{\prod_{i=1}^n s_i^{-a} \frac{1}{S} (n-1)! \frac{(1-a)^n}{na-a+1} (S^{na-n+1} - s_n^{na-a+1})},$$

which simplifies to:

$$B(\mathbf{s}) = \frac{n(1-a)-1}{(S/s_n)^{n(1-a)-1} - 1}.$$

This last equation simplifies to the constant sighting rate case for $a = 0$.

References

Cox, D.R. & Lewis, P.A.W. (1966) *The Statistical Analysis of Series of Events* Methuen, London.

Solow, A.R. (1993) Inferring extinction from sighting data. *Ecology*, **74**(3), 962-64.

Appendix B. Additional figures

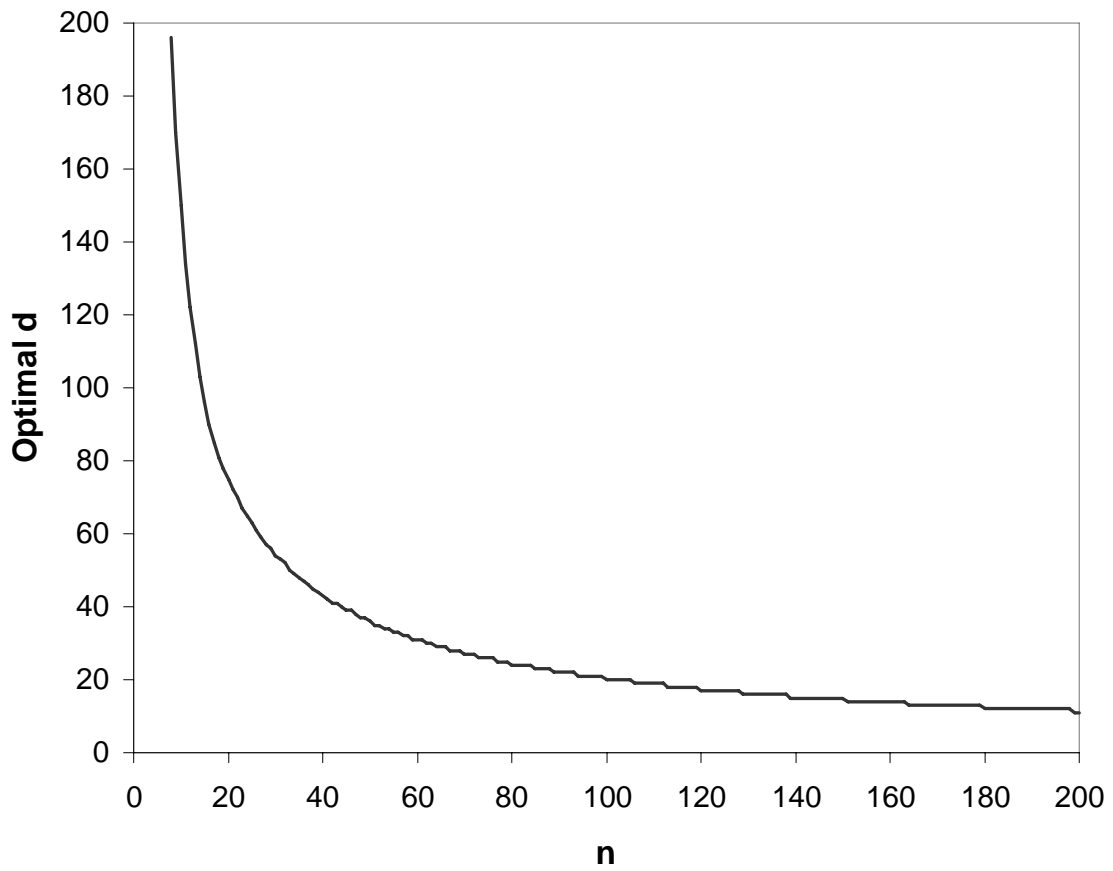
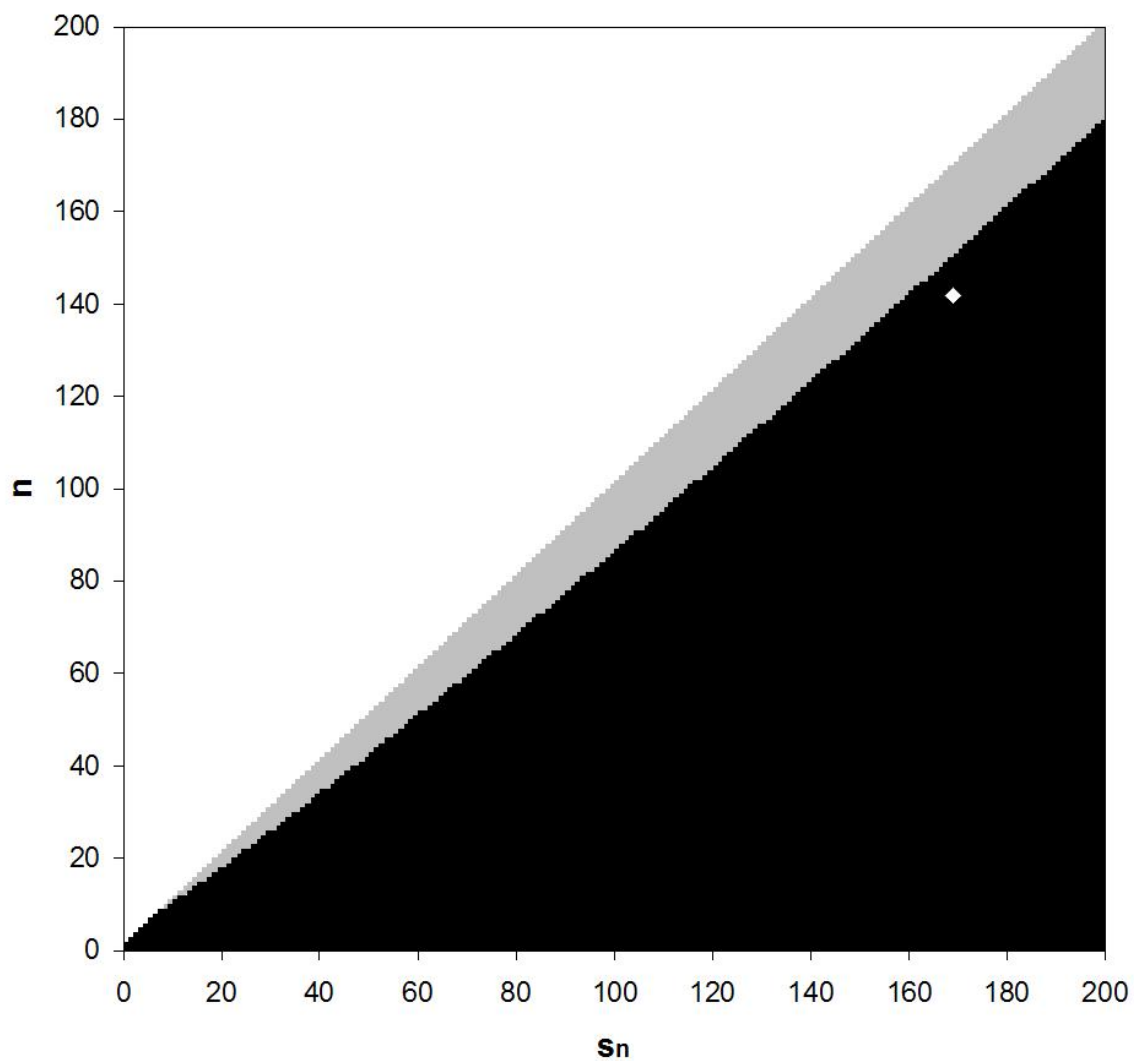


Figure S1: Optimal number of absent surveys before stopping for $s_n = 200$, calculated with the SDP.

a)



b)

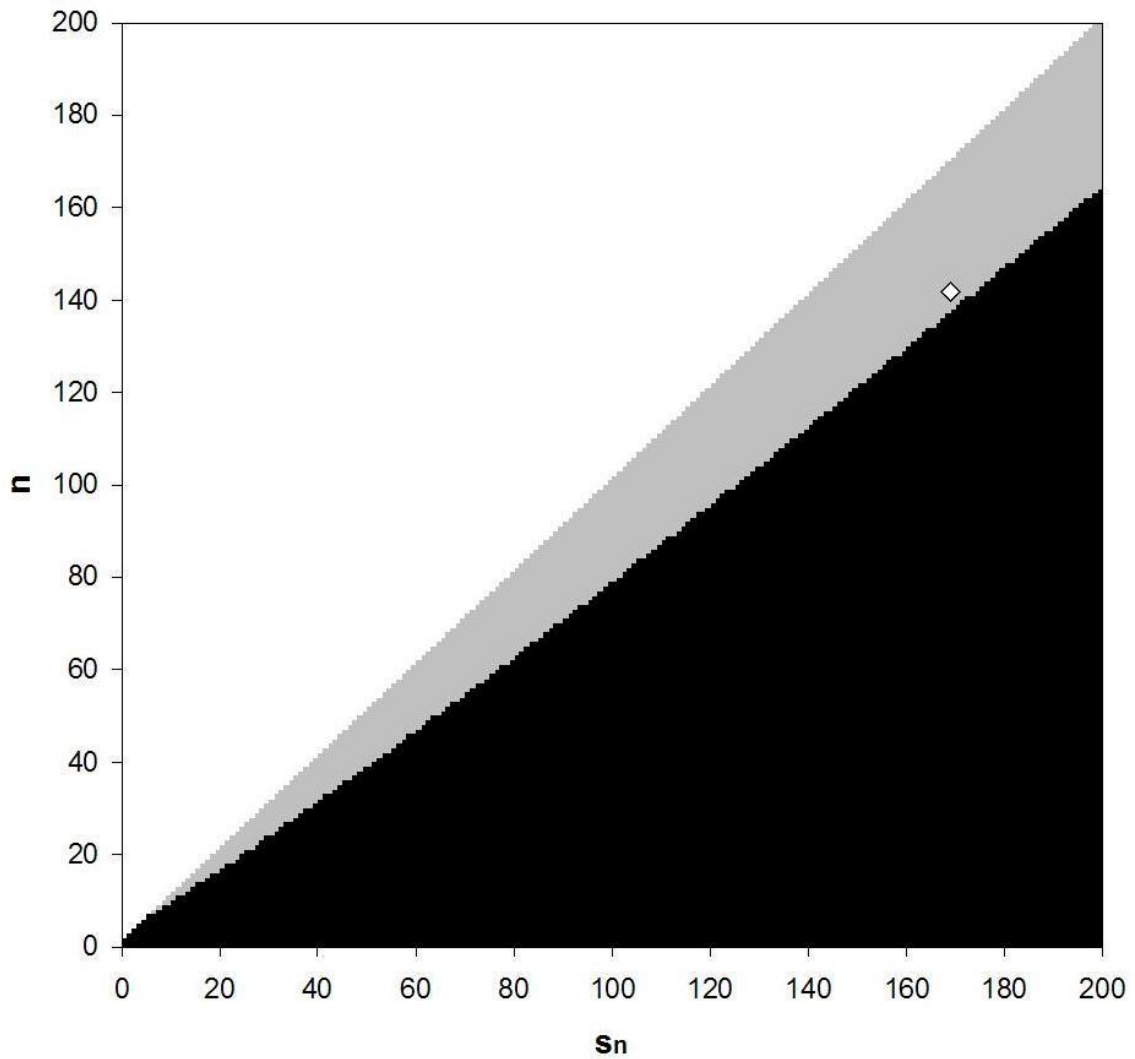


Figure S2: Optimal decisions found with the SDP, using the costs for *H. amarum*. The optimal decision for *H. amarum* is at $s_n=169$, $n=142$, indicated with the white diamond. Black: optimal decision is to keep monitoring, grey: optimal decision is to stop.

a) After 12 absent surveys ($d = 12$), optimal decision for *H. amarum* is to keep surveying.

b) After 13 absent surveys ($d = 13$), optimal decision for *H. amarum* is to stop.