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Bayesian Learning and Synthesis through the Elicitation of Risk: BLASTER

Petra Kuhnert and Simon Barry

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Prepared for: Prof. Mark Burgman

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Enquiries should be addressed to:

Petra Kuhnert
CSIRO Mathematics and Informatics and Statistics
Private Bag 2
Glen Osmond SA 5064
Telephone : +61 8 8303 8775
Fax : +61 8 8303 8763
Email : Petra.Kuhnert@csiro.au

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Contents

1 Abstract

Stochastic modelling of incursion risk is typically an iterative process. A model is developed which calculates likelihood of an adverse event based on parameter values. The parameter values are elicited from experts and Monte Carlo methods used to calculate the likelihood based on the model. In practice this is often an iterative process. The experts consider the likelihood generated by the model and their inputs and assess how consistent it is with their views. If it is inconsistent they can change their beliefs about parameter values, change their views about the incursion risk or even change the model.

We propose a new methodology to help support this process. In the new methodology we also formally elicit beliefs about the overall likelihood. We then provide a principled analysis of the consistency between the inputs and outputs. This will provide structure to the current ad hoc approaches, and will aid in the development and critiquing of models.

The approach simulates from bivariate conditional distributions formed from the input and output parameters given the model, which can converge to the required joint distribution and overcomes the issues noted in previous applications that lead to the Borel paradox. The approach also examines the uncertainties resulting from the model and has the potential to highlight dependencies (or lack thereof) associated with the output and each of the inputs. We illustrate the approach using a simple example and simulation and apply the approach to a case study examining the importation of mangoes in Australia and the risk associated with the establishment of the mango seed weevil in Australian orchards.

Keywords: Expert Opinion; Inconsistency; Markov chain Monte Carlo; Prior probability distributions; Uncertainty

2 INTRODUCTION

Risk analysis involves the quantification of the likelihood and consequence of identified events that may occur at some time in the future. A complex but important issue in this process is quantifying the uncertainty associated with the likelihood and consequence estimates. This can be difficult because little may be known about the processes that may lead to the event. Expert opinion can offer a means for quantifying this information. However, due to the subjectivity and often biased nature of the information, it can be problematic to incorporate it in risk modelling in a transparent manner.

An often used identity in import risk assessments is the incursion equation, $I = 1 - (1 - p)^N$, which defines the overall probability of an incursion, I, via a geometric distribution that takes into account the time or amount of material arriving from an overseas destination, N and the overall probability that the event occurs per unit of material, p, given the sequence of events that led to the invasion. Typically there is little or no data on each of the inputs p and N as well as the output, I. As a result, expert opinion is often used to elicit prior probability distributions for each of these parameters with the evaluation of I being done using the equation and Monte Carlo methods. The opinions elicited about each parameter can be biased, the opinions can vary depending on the choice of experts and the elicitation of p itself, can be poorly conditioned. Furthermore, the evaluation of the left hand side of this equation may be quite different to the result obtained for I using elicitation techniques.

Three techniques that can be used to analyse data arising from import risk assessments are scenario trees (Shen & Zhang 2008, Sherali et al. 2008, Meloy 2004), pathway analysis (McCarthy et al. 2007, Cave et al. 1997) and Bayesian belief networks (BBNs) (Small 2008, Bonafede & Giudici 2006). These types of approaches are similar in terms of how they are structured, i.e. *identifying a pathway of events that lead to an invasion or risk*, but differ in terms of how they are evaluated and how they incorporate uncertainty on the inputs and propagate this to the output of interest. While some use Monte Carlo methods to approximate the propagation of probabilities through a pathway (Vose 1996), others use Bayes theorem to evaluate the posterior probabilities for each node in the model (Lauritzen & Spiegelhalter 1988). More recently, Bates et al. (2003) adopted the Bayesian melding approach of Poole & Raftery (2000) that accounts for uncertainty in the model inputs of a deterministic model for risk and propagates this uncertainty to the outputs. In this application however Bates et al. (2003) made use of experimental data collected on the inputs and outputs of a multi-compartment model and expert opinion elicited about the inputs only. No expert opinion was elicited about the output parameter of interest.

Popular approaches to risk assessment in biosecurity applications typically only consider the forward propagation of uncertainty in the model. In terms of the incursion equation, the parameters p and N may be elicited with the model providing the logical link to infer the consistent beliefs about I. Experience of modelling exercises suggest an iterative approach is common. Experts give their view about parameter values. These are used to estimate the overall incursion risk via the model. The experts consider this risk and assess how consistent it is with their views. If it is inconsistent they can change their belief about parameter values, change their views about the incursion risk or even change the model.

We propose a new methodology to help support this process. In the new methodology we also formally elicit beliefs about I. We then provide a principled analysis of the consistency between the inputs and outputs. This will provide structure to the current ad hoc approaches, and will aid in the development and critiquing of models.

Using the Bayesian synthesis methodology proposed by Raftery et al. (1995), we outline a model that makes use of only expert opinion on the inputs and outputs for risk scenarios. We propose an alternative method for estimation where we simulate from the joint posterior distribution which overcomes issues of inconsistency related to the Borel paradox (Wolpert 1995). The primary advantage of this approach from a risk perspective is its ability to incorporate uncertainty in the inputs and outputs such that, if the expert inputs and outputs are inconsistent, this will be conveyed by the posterior distributions and their dependence amongst one another. This represents an advance in import risk assessments because existing methods ignore information on I and therefore ignore the potential dependence (or lack thereof) associated with the responses to the inputs and outputs. This method also has the ability to identify issues with the elicitation approach used to evaluate risk and has the potential to highlight conditioning problems that occur through the evaluation of p.

Section 3 outlines the Bayesian synthesis methodology in the context of import risk assessments in greater detail and presents a working example and simulation under the assumption of a simple linear constraint in Section 4, where we explore the concept of dependence and how this relates to the input and output parameters of the model. Section 5 then applies the methodology to evaluate the risk of importing mangoes infested with the mango seed weevil into Australia. A discussion of the results and their application to import risk problems more generally is presented in Section 6.

3 METHODS

We propose a Bayesian synthesis approach for risk assessments where the probability of incursion is defined through a deterministic model, M, which takes a suite of inputs, θ, and maps them to an output, ϕ such that

$$M : \theta \rightarrow \phi \text{ and } \phi = M(\theta) \tag{3.1}$$

Using the notation set out by Raftery et al. (1995), we can write down the posterior distribution as

$$\pi^{[\theta]}(\theta) \propto q_1(\theta)L_1(\theta)L_2(M(\theta))q_2(M(\theta)) \tag{3.2}$$

which is in terms of the prior distribution of the inputs, $q_1(\theta)$ and output, $q_2^*(\phi) = q_2(M(\theta))$, and likelihoods prescribed for the inputs, $L_1(\theta)$ and output, $L_2(\phi) = L_2(M(\theta))$. In situations where no data exists for θ or ϕ, the posterior distribution shown in Equation 2 reduces to prior distributions on θ and ϕ.

3.1 Implementation and simulation

Raftery et al. (1995) and Poole & Raftery (2000) promote the Sampling Importance Resampling (SIR) algorithm of Rubin (1988) for the estimation of parameters from this model. This approach draws a large sample of input values, θ from the respective priors and then evaluates ϕ through the deterministic equation, M and the relationship shown in Equation 1. As highlighted by Poole & Raftery (2000), if a prior is placed on the output parameter then there is the potential for inconsistencies between the prior distribution on the output, $q_2(\phi)$ and the induced distribution, $q_2^*(\phi) = M(\theta)$ and in fact, any parameterisation of it. This is known as the Borel paradox (Wolpert 1995). To overcome this problem and ensure that the conditional probability densities are invariant with respect to reparameterisations of the model, Poole & Raftery (2000) proposed pooling the prior probability distributions placed on the inputs and output using geometric pooling approaches.

We propose an alternative approach to simulation which avoids the sometimes complicated task of pooling priors. The approach considers using Markov Chain Monte Carlo (MCMC) to draw samples from bivariate conditional distributions, which are then used to approximate the required distribution. For example, consider three random input parameters, x_1, x_2 and x_3, which are constrained by the linear equation, $x_1 + x_2 + x_3 = x_4$ where x_4 represents the output variable of interest. Assuming that we have formed prior distributions $f(x_1), \dots, f(x_4)$ through an elicitation exercise with relevant experts, we can use this information to make tighter inference about the true underlying values and refine our estimates of uncertainty by forming the joint prior beliefs as,

$$[x_1, x_2, x_3, x_4] \propto f(x_1)f(x_2)f(x_3)f(x_4)$$

Using MCMC, we can simulate from this distribution using the following conditional distributions, where [ ] represents a distribution of interest,

$$[x_1|x_2, x_3, x_4][x_2|x_1, x_3, x_4][x_3|x_1, x_2, x_4][x_4|x_1, x_2, x_3]$$

with each variable updated in turn. Mathematically, it can be shown that the set $\{x_1, x_2, x_3, x_4\}$ will converge in distribution to the joint distribution of the four variables,$[x_1, x_2, x_3, x_4]$. In particular, empirical function means based on the sequence converge with probability one under mild regularity conditions, i.e.

$$\sum_i f(x_1, x_2, x_3, x_4) \to \int f(x_1, x_2, x_3, x_4)[x_1, x_2, x_3, x_4]dx_1dx_2dx_3dx_4$$

In our case, assume we learn that the constraint, $x_1+x_2+x_3 = x_4$, applies to our variables. We could attempt to use simple univariate conditional distributions but these are degenerate. For example $[x_1, |x_2, x_3, x_4]$ takes the value $x_4 - x_2 - x_3$ with probability one. We therefore need to consider bivariate conditionals

$$[x_1, x_2|x_3, x_4][x_2, x_3|x_1, x_4][x_3, x_4|x_1, x_2] \tag{3.3}$$

which can be shown to converge to the required joint distribution subject to the constraint. For example, $[x_1, x_2|x_3, x_4]$ has the constraint $x_1 = x_4 - x_3 - x_2$ and the probability distribution for the draw would need to occur along this line.

As a simple example let x_1, x_2, x_3 and x_4 be assigned standard Gaussian distributions. In this example, the constraint $x_1 + x_2 + x_3 = x_4$ is inconsistent with this specification of the priors as the summation $(x_1 + x_2 + x_3)$ will have variance 3 while the variance of x_4 is one. If we structure this problem in terms of bivariate conditional distributions as shown in Equation 3.3 we have the following conditional distribution for x_1 and x_2,

$$[x_1, x_2|x_3, x_4] \propto \exp -\frac{1}{2}x_1^2 \exp -\frac{1}{2}x_2^2 \tag{3.4}$$

with constraint $x_1 = x_4 - x_3 - x_2$. Letting $k_1 = x_4 - x_3$, x_1 becomes $k_1 - x_2$ and substituting this into Equation 3.4 results in the expression,

$$\begin{aligned}[x_2|k_1] &\propto \exp[-\frac{\gamma}{2}(x_2 - \frac{k_1}{2})^2] \\ &\sim \mathrm{N}(\frac{k_1}{2}, \sqrt{\gamma^{-1}})\end{aligned} \tag{3.5}$$

where $\gamma = 2$ and represents the precision. We can then simulate x_2 directly from this conditional distribution given k_1 and solve for x_1 and repeat this process for the remaining bivariate conditional distributions. See Appendix A for the derivation of all of the bivariate conditional distributions for the general case. The algorithm for the simulation approach is shown in Algorithm 1.

Algorithm 1 Simulating the joint distribution

- Set initial values for x_3 and x_4.
for $i = 1$ to N iter **do**
  - Simulate x_2 from $[x_2|x_3, x_4]$
  - Evaluate x_1 using the expression $x_1 = x_4 - x_3 - x_2$
  - Simulate x_3 from $[x_3|x_1, x_4]$
  - Evaluate x_2 using the expression $x_2 = x_4 - x_1 - x_3$
  - Simulate x_4 from $[x_4|x_1, x_2]$
  - Evaluate x_3 using the expression $x_3 = x_4 - x_1 - x_2$
end for
- Evaluate convergence for $x_1, \ldots, x_4$.

The results of this sampler for a run length 10,000 are shown in Figure 6.1, where a pairwise plot of the simulated parameters is displayed. Convergence was assessed using the diagnostics outlined in Cowles & Carlin (1996). The variance-covariance matrix for the posterior distribution of parameters is represented as

	x1	x2	x3	x4
x1	0.7629666	-0.2197641	-0.2834934	0.2597090
x2	-0.2197641	0.7316212	-0.2672342	0.2446229
x3	-0.2834934	-0.2672342	0.7894797	0.2387520
x4	0.2597090	0.2446229	0.2387520	0.7430839

In this matrix, we see that the correlation is induced from the dependency relationships between the parameters and the variance is reduced to satisfy the logical requirements of the constraint. Most importantly, the variance of the sum of x_1, x_2 and x_3 is equal to 0.7430839, which is consistent with the variance of x_4 in the posterior sample.

4 SIMULATION STUDY

We constructed six prior scenarios to investigate the behaviour of the posterior distribution of the parameters x_1, x_2, x_3 and x_4 in the example above. The scenarios are summarised in Table 6.1 and differ according to the mean and standard deviation assigned to each prior. Our aim in these scenarios was to investigate how changes to the prior placed on one input variable, (x_1), impacted the posterior distribution of the remaining input variables as well as the output.

The first scenario in our investigation mimics the example presented in Section 3 and assumes that all priors have a mean of 0 and a standard deviation of 1. Scenarios 2-4 investigate altering the prior for x_1. In each of these scenarios the mean is set to 5, while the standard deviation varies from non-informative ($\sigma = 100$), moderately informative ($\sigma = 1$) to very informative ($\sigma = 0.1$). A non-informative scenario is investigated in Scenario 5, while an informative prior scenario, where the prior mean for x_1 is equal to 5, is set for Scenario 6. The results from running each of these scenarios is displayed in Figures 6.2 and 6.3. Note the different scales used for the axes of each plot.

Scenario 1 shows little difference between the prior and posterior distributions of the parameters in the model (Figure 6.2(a)). This has resulted from making all of the priors equal in the model, therefore resulting in posterior distributions which are mildly dependent on each other (Figure 6.2(b)) such that the constraint is satisfied. A similar result occurred for Scenario 5, where the priors for all of the parameters in the model were non-informative with the same mean and standard deviation. See Figures 6.3(c) and 6.3(d). The strong prior information investigated through Scenario 6 leads to posterior distributions that are slightly different to the initial prior set (Figure 6.3(e)). The differences between the prior and posterior for each parameter are the result of the constraint set up in the model. This results in the strong dependent nature of the parameters as shown in Figure 6.3(f).

Overall, we see that as the prior for x_1 becomes more informative, the dependency between the parameters x_2 through to x_4 becomes stronger, indicating higher correlations between these parameters. We also observe that the dependency between x_1 and the remaining parameters is close to zero as illustrated by the spherical nature of the plots shown in Figure 6.2(f). When the prior for x_1 is non-informative, as shown in Figure 6.3(a), we see the opposite relationships occurring. The correlation between x_1 and the remaining parameters appears high while low correlations are apparent for the remaining parameters, as indicated by the spherical shape of these pairwise plots. When the prior is moderately informative, as in the case of Scenario 2, we see mild correlation between all parameters in the model. These results are intuitive because when the prior for a particular variable is informative, its resulting posterior distribution will have little or no dependence on the remaining parameters in the model. However, the posterior distributions of the remaining parameters themselves will be dependent because they will need to satisfy not only the constraint set by the problem but the informative prior set for that one parameter, which for this example is x_1. If the situation is reversed and one of the variables is non-informative while the remaining variables are moderately informative, we find the opposite effect. Now, the posterior distribution for the non-informative variable is dependent on the other parameters in the model as well as the constraint.

If we focus our attention on making dramatic changes to two or more parameters in the model, say x_1, x_2 and x_3 we find some interesting results. Figure 6.4(a) and 6.4(b) summarise the results from fitting an informative prior to x_1 and x_2 ($\sim N(5, 0.1)$) and standard Gaussian priors to x_3 and x_4 (Scenario 7 - Table 6.1). The pairwise plots show little correlation between x_1 and x_2 with the remaining parameters in the model and this is also emphasised by the plot in Figure 6.4(a), where both the prior and posterior distributions for x_1 and x_2 are virtually the same. However, there is strong correlation demonstrated between x_3 and x_4, which is expected, since conditional on the priors for x_1 and x_2, x_3 and x_4 are required to satisfy the constraint and since the respective priors for these parameters are the same, the correlation is strong. We found that the more non-informative we make these priors, the stronger the correlation.

The results from investigating Scenario 8 is presented in Figures 6.4(c) and 6.4(d). This scenario explores informative priors ($\sim N(5, 0.1)$) for x_1, x_2 and x_3 and a standard Gaussian prior for x_4. As expected, the results indicate some dependency between x_4 and the remaining parameters in the model which becomes stronger the more non-informative the prior for x_4 is. Due to the informative nature of the priors for x_1, x_2 and x_3, there is little or no correlation between these variables. When we compare the prior and posterior distributions for this scenario, the posterior distributions differ markedly. This is the result of the informative nature of three of the prior distributions and the constraint imposed on the modelling problem.

From an elicitation perspective, these plots are useful diagnostic measures as they help the elicitor and expert understand the resulting output and they can guide the expert into revising their opinion about a particular parameter if the posterior output is inconsistent with their original opinion.

5 CASE STUDY IN IMPORT RISK ASSESSMENT

We now consider a simple risk-based model consisting of the probability or likelihood of an event. A well established result in import risk assessments is Equation 5.1, which defines the overall probability of an incursion I, via a geometric distribution that takes into account the time or amount of material arriving from an overseas destination, N and the overall probability that the event occurs, p given the sequence of events that led to the invasion.

$$I = 1 - (1 - p)^N \tag{5.1}$$

If we assume that the input parameters p and N are known with some uncertainty we can define prior distributions for these parameters, $f(p)$ and $f(N)$. Furthermore, if we can elicit the expert's a priori belief about the outcome parameter, I, we can estimate the joint prior beliefs, $[N, p, I]$ as the product of $f(N)$, $f(p)$ and $f(I)$ with a constraint defined through Equation 5.1. For this example the constraint represents a curved surface in the 3-dimensional space spanned by the three parameters. We propose finding the conditional or posterior distribution of the input parameters that satisfies this constraint.

5.1 Construction of Bivariate Conditional Distributions

We consider taking a transformation of the parameters in the model shown in Equation 6 to assist with convergence and assign normal distributions to each such that,

$$\begin{aligned} [N' = \log(N)] &\sim \mathrm{N}(\mu_{N'}, \sigma_{N'}) \\ [p' = \mathrm{logit}(p)] &\sim \mathrm{N}(\mu_{p'}, \sigma_{p'}) \\ [I' = \mathrm{logit}(I)] &\sim \mathrm{N}(\mu_{I'}, \sigma_{I'}) \end{aligned} \tag{5.2}$$

Considering bivariate conditional distributions for the parameters in the model we have

$$[N', p'|I'][p', I'|N']$$

where

$$
\begin{aligned}
[N', p'|I'] &\propto \exp\{-\frac{1}{2\sigma^2_{N'}}(N' - \mu_{N'})^2\} \exp\{-\frac{1}{2\sigma^2_{p'}}(p' - \mu_{p'})^2\} \\
[p', I'|N'] &\propto \exp\{-\frac{1}{2\sigma^2_{p'}}(p' - \mu_{p'})^2\} \exp\{-\frac{1}{2\sigma^2_{I'}}(I' - \mu_{I'})^2\}
\end{aligned}
\tag{5.3}
$$

Given the constraint specified in Equation 5.1 and the bivariate conditional distributions defined above we can now form posterior distributions for p, I and N and simulate using Metropolis algorithms. Simulations were conducted using the Winbugs package with a 100000 burn-in following by a further 100000 iterations which were used to make inference. Hyper-parameters chosen for the prior distributions were based on an elicitation conducted with experts, the method of which has been described elsewhere (Speirs-Bridge et al. 2009). This resulted in the following prior distributions and indicates immediately some inconsistencies in the specification of parameters for the right hand side of the model in Equation 6 and the response, I.

$$
\begin{aligned}
[N'] &\sim \mathrm{N}(\mu_{N'} = 6.477, \sigma_{N'} = 0.788) \\
[p'] &\sim \mathrm{N}(\mu_{p'} = -18.811, \sigma_{p'} = 1.705) \\
[I'] &\sim \mathrm{N}(\mu_{I'} = -4.648, \sigma_{I'} = 2.525)
\end{aligned}
\tag{5.4}
$$

The results from this simulation are displayed in Figure 6.5(a) and show the prior (solid line) and posterior (dotted line) distributional summaries (mean and 95% credible intervals) for each parameter in the model on the transformed scale. The results show a substantial shift in the posterior estimate for I compared to its prior specification, indicating inconsistencies between what was elicited from the experts about I and the evaluation of the right hand side of Equation 5.1. The posterior estimate for p showed a minimal change in the mean but tightening of the 95% credible intervals around that estimate. Minimal changes were observed for N. Figure 6.5(b) shows a pairwise plot of the transformed parameters and shows strong correlation between p and I and little correlation between N and p, and N and I indicating that the probability of incursion is heavily reliant on what the expert knows about p.

To investigate the impact of the expert prior in this model we structured four scenarios. The first two investigate increasing and reducing the variability around p (Scenarios 1 and 2 respectively), while the remaining two examine the variability around the expert opinion for I. Scenario 3 investigates greater variability while Scenario 4 examines less variability in the prior specification for I. Results are displayed in Figure 6.6 and 6.7 for the four scenarios. Figure 6.6 shows the prior and posterior distributions for each parameter in the model after implementing the four scenarios. Figure 6.7 shows corresponding pairwise plots of the posterior estimates. We summarise the main results from this scenario investigation below.

- If an expert is more uncertain about his/her estimate of p, then introducing this variability around the prior for p makes the posterior estimate for I more consistent with its prior (Figure 6.6(a)). As a result, the posterior mean estimate for p is shifted towards I with tighter credible intervals to satisfy the constraint. The dependent nature of I and p is illustrated in Figure 6.7(a). This has little impact for N as indicated by the pairwise plots showing a lack of dependence between N and the other parameters in the model.
- If an expert was more confident in their prior estimate for p, we found that the posterior estimate for I depended somewhat on p but more so on N as shown in Figure 6.7(b). This

resulted in a very tight credible interval for the posterior estimate of p not too different from the prior, a posterior mean estimate for I shifted towards p and a slight shift in the posterior mean estimate for N to satisfy the constraint (Figure 6.6(b)).

- Scenario 3 showed similar dependency structures to Scenario 1 (Figure 6.7(c)). Introducing more variability around I (from an expert that is uncertain about the parameter) results in a posterior estimate for I that is largely dependent on p and has little or no bearing on the posterior distribution for N (Figure 6.6(c)).
- Altering the prior for I so it is less variable and therefore much tighter (reflecting an expert's confidence) results in slight dependencies between I and p and I and N (Figure 6.7(d)). This is reflected by the shift in distribution for p and N as shown in Figure 6.6(d)

Based on the prior information provided for all three parameters, there appears to be some obvious inconsistencies between the specified priors for I and p and posterior distributions as identified by the mango weevil problem. The model appears to be largely driven by the priors for I and p as seen through the scenario investigation, with estimates for N that are dictated by these prior specifications and the constraint set up in the model. Table 6.2 displays the results for each parameter in the model on the transformed and raw scale and highlight an incursion probability of 1.032×10^{-4}, CI:[8.593×10^{-6}, 4.376×10^{-4}] (nearly a 1 in 10000 chance) of observing an incursion from the mango seed weevil given that the probability of an infestation occurring is small (6.5×10^{-8}, CI=[4.15×10^{-9},2.86×10^{-7}]) and the amount of mango boxes arriving is approximately 1194, CI:[219, 3839].

6 DISCUSSION

We have presented a Bayesian approach for synthesising expert information conditional on some underlying model of interest. The approach is based on the Bayesian synthesis approach of Raftery et al. (1995) but implements an alternative method for simulation that overcomes the issues noted by Poole & Raftery (2000) due to the Borel paradox. Instead of simulating from conditional distributions in the usual way, we simulate from bivariate conditional distributions formed from the parameters of interest subject to a constraint defined via the model. This ensures consistency between the input priors and output priors defining the model and induces a correlation between parameters that can be useful in determining relationships between parameters and, in particular, the impact each prior has on other parameters in the model. We illustrated this using a simple example where standard normal priors were chosen for four parameters subject to a linear constraint. We examined 7 scenarios and showed that when a prior for a particular parameter is informative, the resulting posterior distribution will have little or no dependence on the remaining parameters in the model. However, the remaining parameters will exhibit dependence.

The results from applying this methodology to a real problem in import risk assessment show inconsistencies between I and p in the model as the priors and posterior distributions change considerably. Furthermore, the posterior estimates from the model show strong dependence such that, if a change occurs in the prior specification of one parameter, it has a strong effect on the remaining parameters in the model. We illustrated this through four examples, where we altered the prior specification for p to reflect higher or lower variability (Figures 6.6(a) and 6.6(b)). The results indicated that as the variability of the prior surrounding p increased, the less influence this prior had on I or N (Figure 6.6(a)). This is also true for I. If the prior specification for I was more uncertain, the posterior distribution for I became influenced by the prior for p (Figure 6.6(c)). When we structured a more informative prior for p or I we found that these priors influenced the remaining parameters in the model in order to satisfy the constraint (Figures 6.6(b) and 6.6(d)).

This application of Bayesian synthesis is an advancement on existing approaches to risk assessment as it highlights inconsistencies in the expert prior specifications and allows the expert to evaluate their opinion in light of the model's posterior estimates. In addition it produces a consistent analysis that is closest, in terms of prior densities, to the elicited values. Given the potentially biased nature of expert opinion and the lack of empirical data to validate expert responses, particularly about risk, it is important to have a transparent way of incorporating expert opinion into a risk assessment and be able to challenge its credibility. In the case of the mango seed weevil, we would encourage experts to have a discussion around the responses that they provided in the assessment to determine whether they agree with the posterior estimates produced from the model, or whether their initial prior assessments were credible in the first place.

The subjective nature of expert opinion warrants further scrutiny of elicitation practices and the priors that are produced and incorporated into risk assessments. Too often, expert information is used without much thought or rigor simply because empirical data is lacking and expert status is regarded as a good substitute. As highlighted by Kuhnert et al. (2010), experts can provide an inaccurate assessment without any intention. Validation of elicited quantities is therefore imperative in any type of risk assessment. We believe the approach presented here provides a mechanism for challenging these opinions in risk assessments where there is no data and identify where inconsistencies arise and whether further investigations of input parameters in a model need to be conducted.

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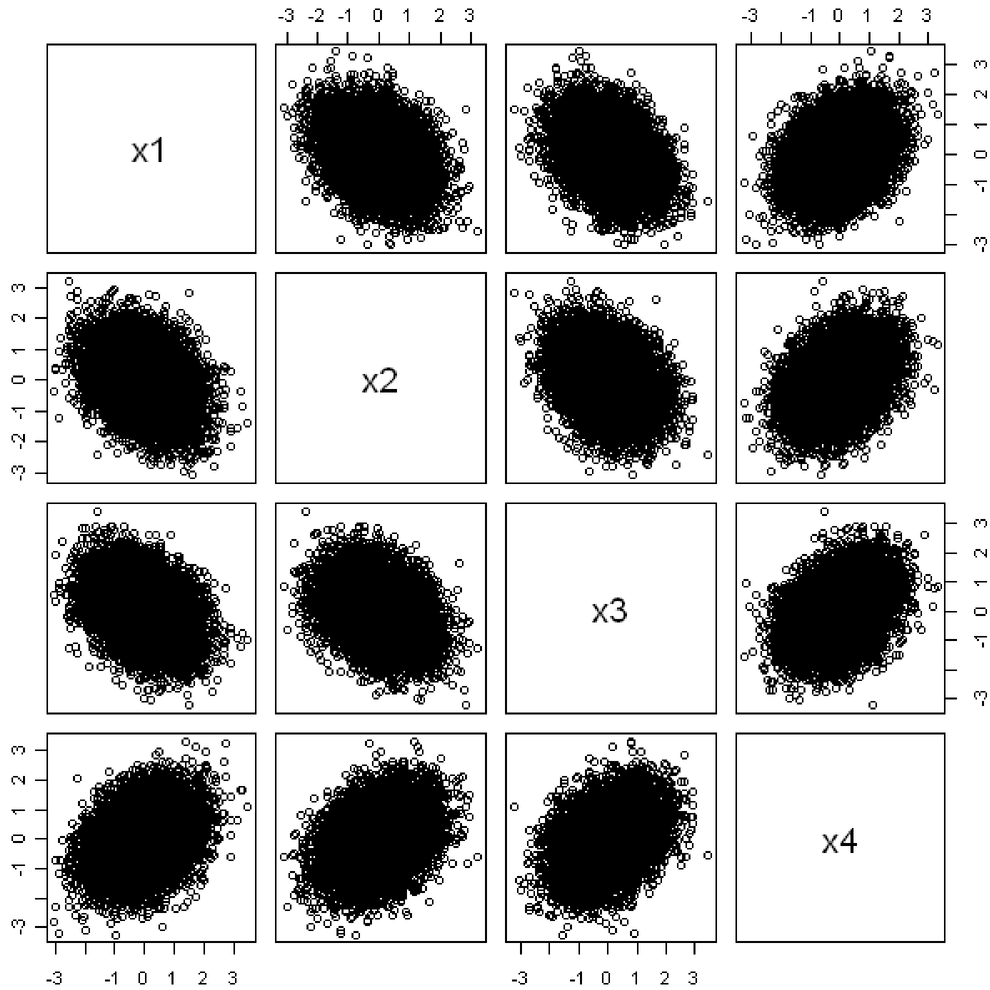
List of Figures

Figures



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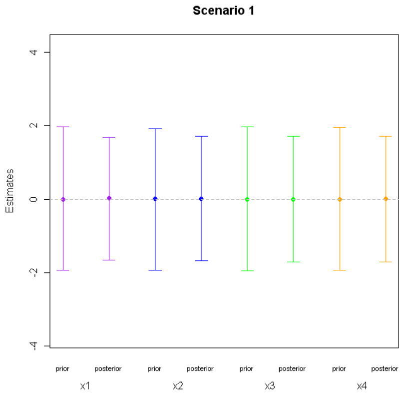

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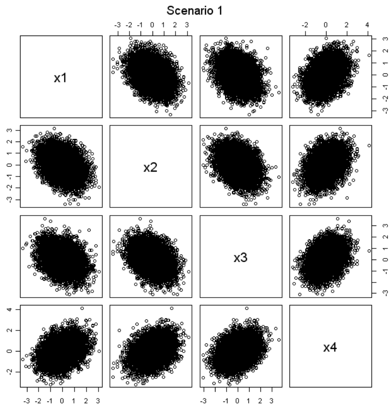

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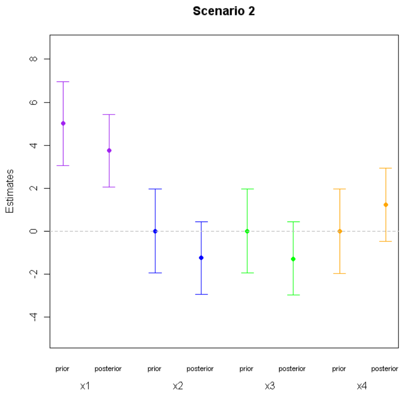

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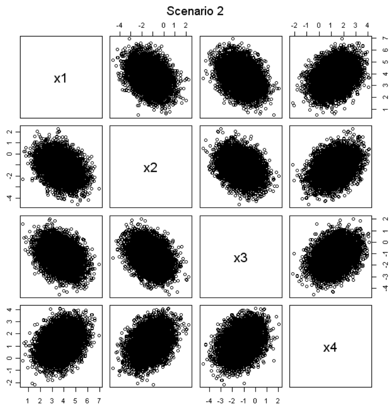

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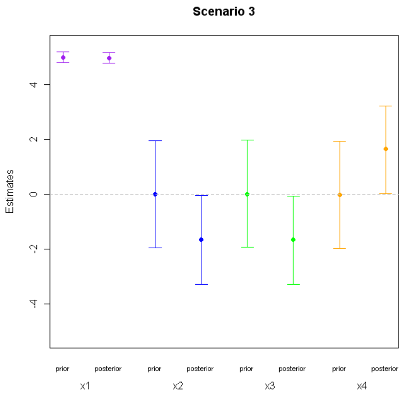

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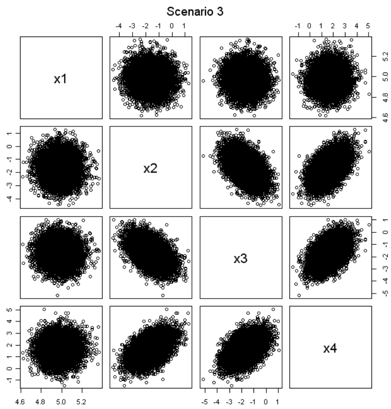

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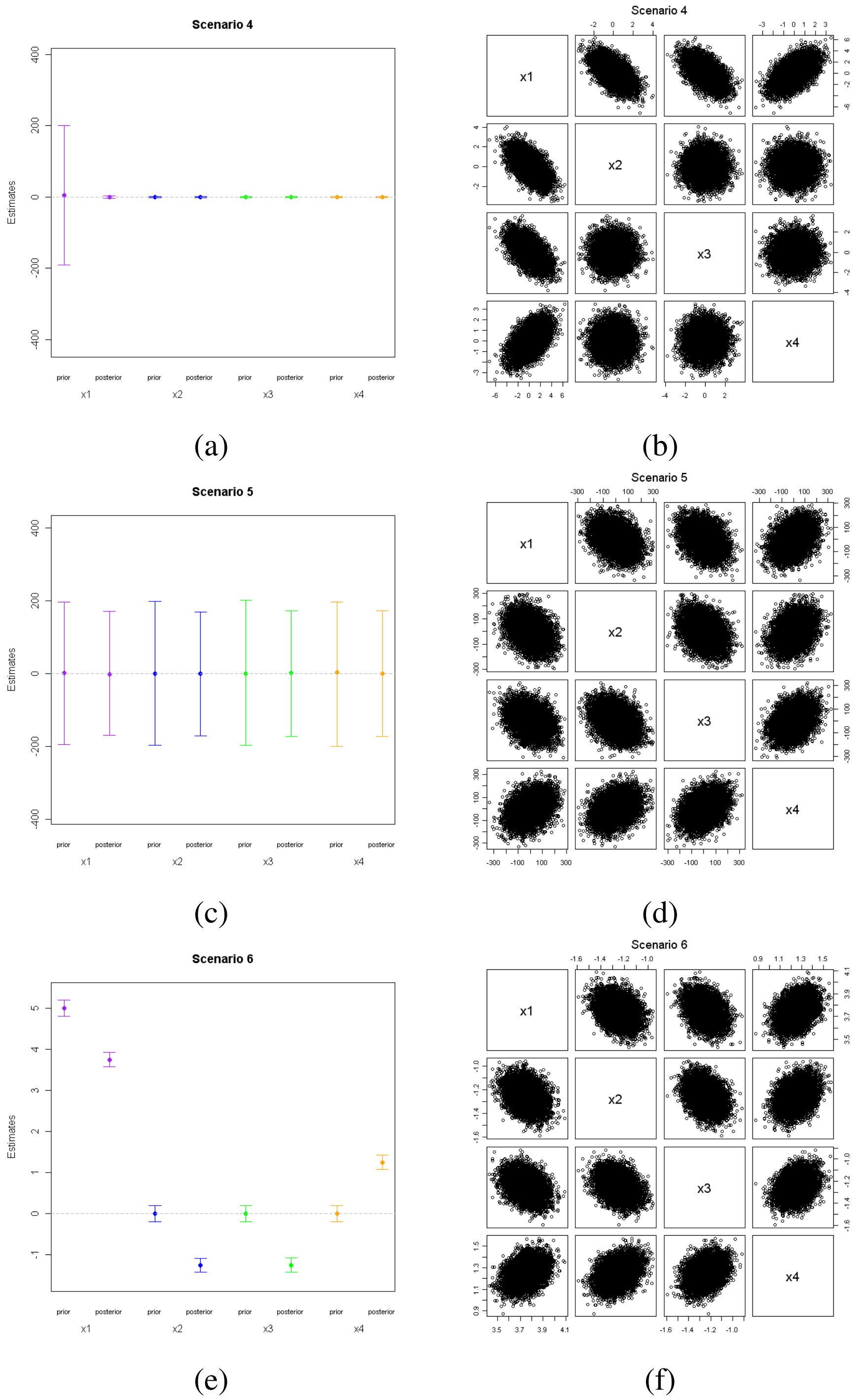
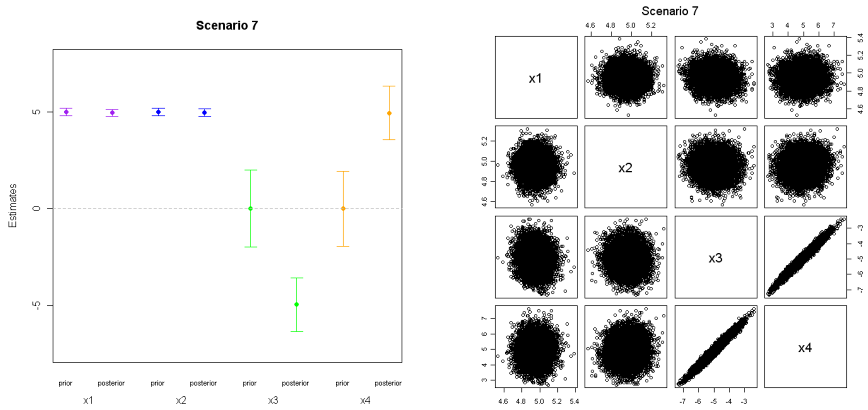
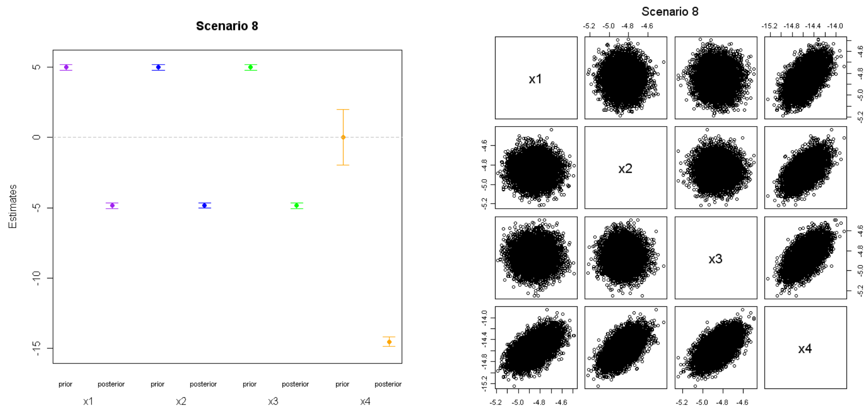

Scenario 4
Estimates
prior
posterior
x1
x2
x3
x4
(a)
(b)
Scenario 5
(c)
(d)
Scenario 6
(e)
(f)

Figure 6.3:



(a) (b)



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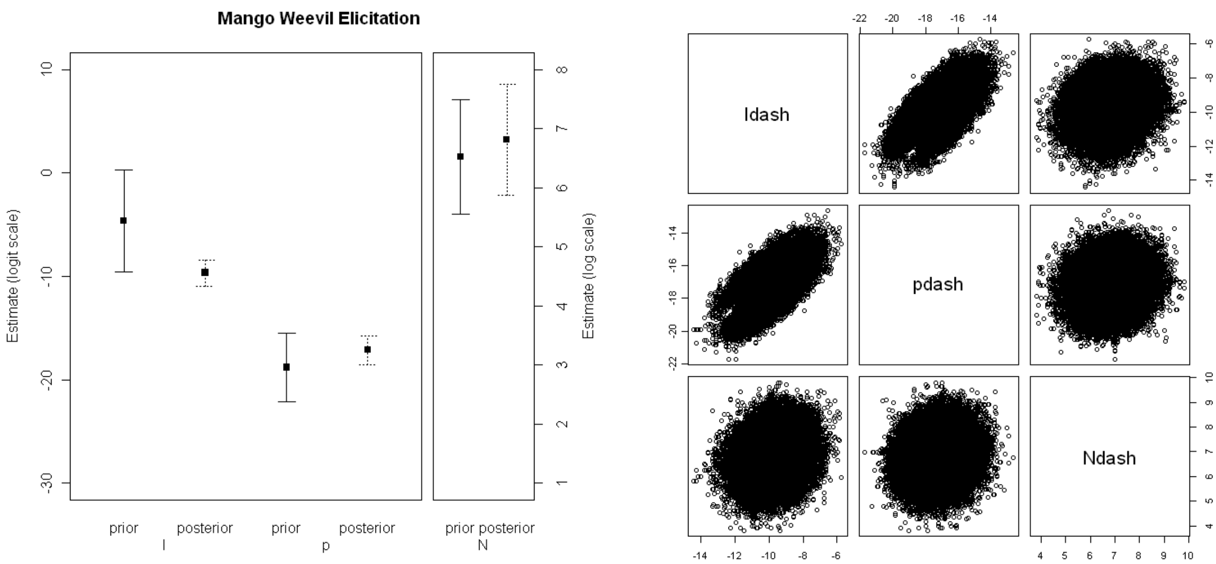

(a) (b)

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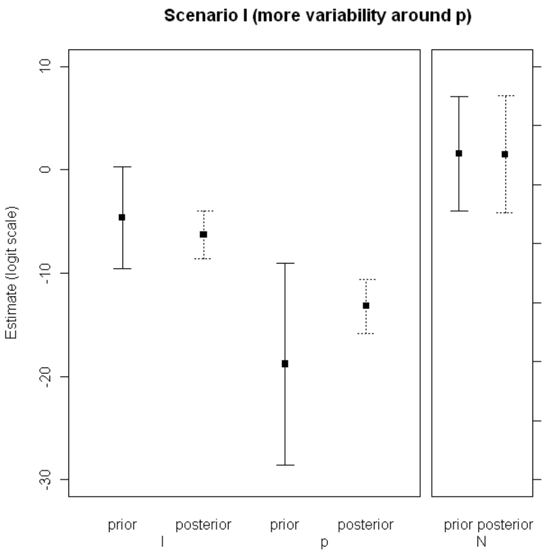

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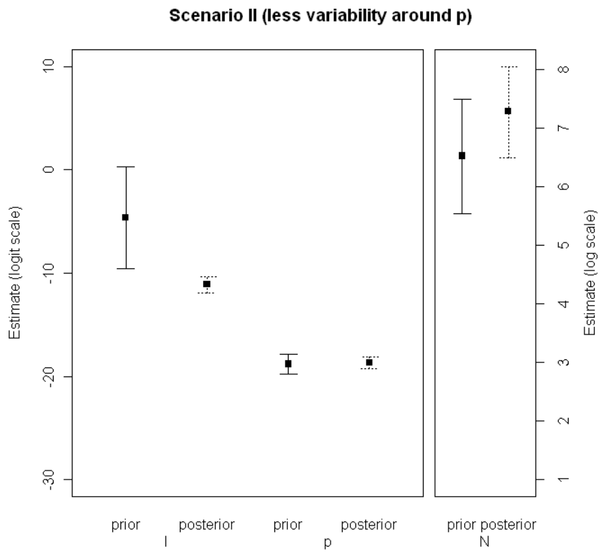

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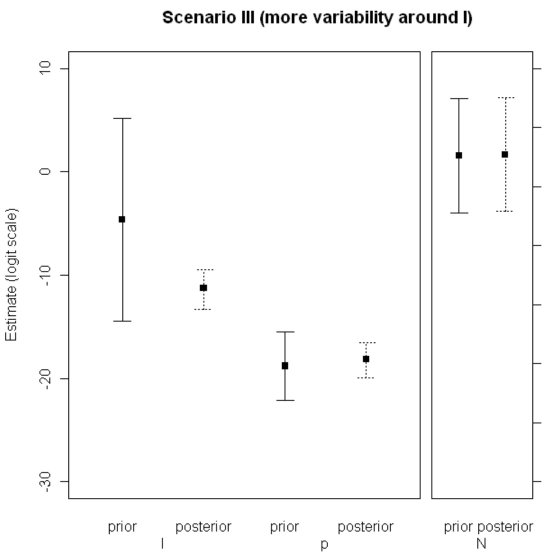

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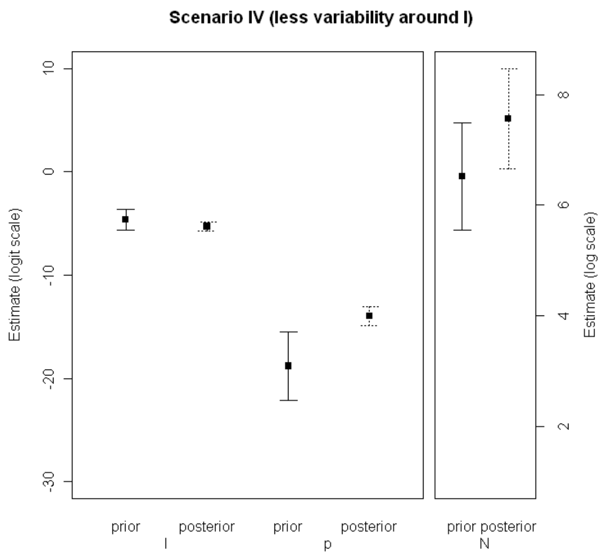

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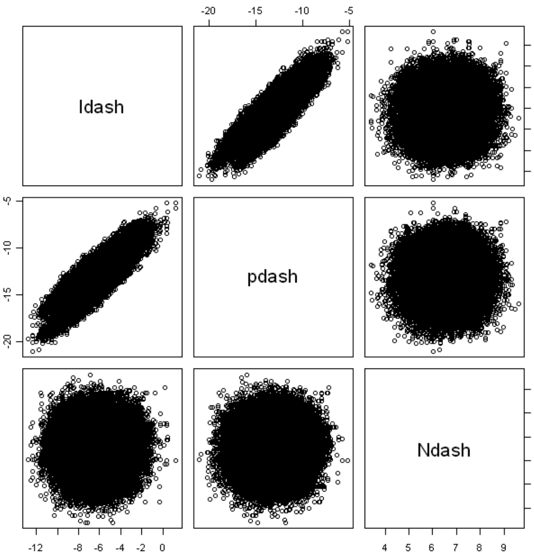

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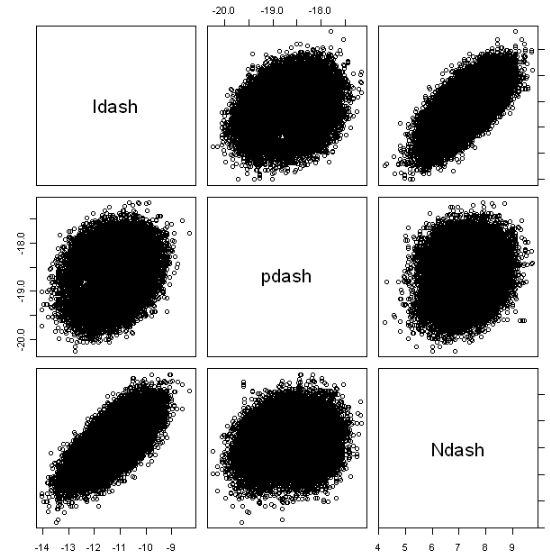

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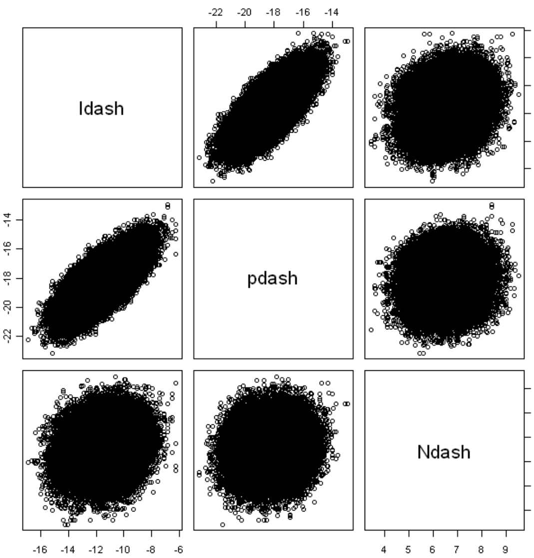

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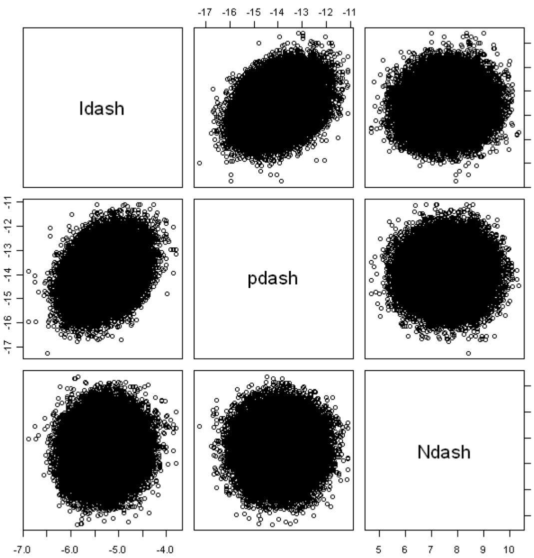

(b)

Figure 6.7:

Tables

Table 6.1: Summary of scenarios investigated for the linear constraint example.

Scenario	Distribution for				Type
	x_1	x_2	x_3	x_4	
1	$N(0, 1)$	$N(0, 1)$	$N(0, 1)$	$N(0, 1)$	All priors equal
2	$N(5, 1)$	$N(0, 1)$	$N(0, 1)$	$N(0, 1)$	Prior: x_1 large μ
3	$N(5, 0.1)$	$N(0, 1)$	$N(0, 1)$	$N(0, 1)$	Prior: x_1 large μ, small σ
4	$N(5, 100)$	$N(0, 1)$	$N(0, 1)$	$N(0, 1)$	Prior for x_1 large μ, large σ
5	$N(0, 100)$	$N(0, 100)$	$N(0, 100)$	$N(0, 100)$	All priors non-informative
6	$N(5, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	All priors informative, x_1 large μ.
7	$N(5, 0.1)$	$N(5, 0.1)$	$N(0, 1)$	$N(0, 1)$	x_1 and x_2 informative.

Table 6.2: Summary of results from fitting BLASTER to the mango weevil data. Estimates shown are the posterior mean, standard deviation (SD) and 95% credible intervals (CI) for the transformed and raw variables.

Parameter	Estimate	SD	95% CI	
			Lower	Upper
I'	-9.663	0.998	-11.66	-7.734
p'	-17.119	1.091	-19.3	-15.07
N'	6.816	0.731	5.391	8.253
I	1.032×10^{-4}	1.278×10^{-4}	8.593×10^{-6}	4.376×10^{-4}
p	6.497×10^{-8}	8.951×10^{-8}	4.149×10^{-9}	2.857×10^{-7}
N	1194	1023	219	3839

A Conditional Distributions for Linear Case

A.1 Simulating $[x_1, x_2|x_3, x_4]$

Consider the conditional distribution of the form

$$[x_1, x_2|x_3, x_4] \propto \exp\{-\frac{1}{2\sigma_1^2}(x_1 - \mu_1)^2\} \exp\{-\frac{1}{2\sigma_2^2}(x_2 - \mu_2)^2\} \tag{A.1}$$

with constraint $x_1 = x_4 - x_3 - x_2$. If we let $k_1 = x_4 - x_3$, the above constraint becomes a function of a constant, k_1 and x_2 only, $x_1 = k_1 - x_2$. Substituting this into Equation A.1 produces

$$\begin{aligned}
[x_2|k_1, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2] &\propto \exp\{-\frac{1}{2\sigma_1^2}(k_1 - x_2 - \mu_1)^2\} \exp\{-\frac{1}{2\sigma_2^2}(x_2 - \mu_2)^2\} \\
&= \exp\{-\frac{1}{2}[\frac{1}{\sigma_1^2}((k_1 - \mu_1)^2 - 2(k_1 - \mu_1)x_2 + x_2^2) + \frac{1}{\sigma_2^2}(x_2^2 - 2\mu_2 x_2 + \mu_2^2)]\} \\
&= \exp\{-\frac{1}{2}[x_2^2(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}) - 2x_2(\frac{k_1 - \mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}) + \frac{(k_1 - \mu_1)^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2}]\} \\
\text{Let } \gamma &= \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} \text{ then} \\
[x_2|k_1, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2] &= \exp\{-\frac{\gamma}{2}[x_2^2 - \frac{2x_2}{\gamma}(\frac{k_1 - \mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}) + \frac{1}{\gamma}(\frac{(k_1 - \mu_1)^2}{\sigma_1^2} + \frac{\mu_2^2}{\sigma_2^2})]\} \\
&\propto \exp\{-\frac{\gamma}{2}[(x_2 - \frac{1}{\gamma}(\frac{k_1 - \mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}))^2]\} \\
&\sim N(\frac{1}{\gamma}[\frac{k_1 - \mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}], \frac{1}{\sqrt{\gamma}}) \\
&= N(\frac{k_1}{2}, \frac{1}{\sqrt{2}}) \text{ when } \mu_1 = \mu_2 = 0 \text{ and } \sigma_1^2 = \sigma_2^2 = 1
\end{aligned} \tag{A.2}$$

A.2 Simulating $[x_2, x_3|x_1, x_4]$

Consider the conditional distribution of the form

$$[x_2, x_3|x_1, x_4] \propto \exp\{-\frac{1}{2\sigma_2^2}(x_2 - \mu_2)^2\} \exp\{-\frac{1}{2\sigma_3^2}(x_3 - \mu_3)^2\} \tag{A.3}$$

with constraint $x_2 = x_4 - x_1 - x_3$. If we let $k_2 = x_4 - x_1$, the above constraint becomes a function of a constant, k_2 and x_3 only, $x_2 = k_2 - x_3$. Substituting this into Equation A.3

produces

$$
\begin{aligned}
[x_3|k_2, \mu_2, \mu_3, \sigma_2^2, \sigma_3^2] &\propto \exp\{-\frac{1}{2\sigma_2^2}(k_2 - x_3 - \mu_2)^2\} \exp\{-\frac{1}{2\sigma_3^2}(x_3 - \mu_3)^2\} \\
&= \exp\{-\frac{1}{2}[\frac{1}{\sigma_2^2}((k_2-\mu_2)^2 - 2(k_2-\mu_2)x_3 + x_3^2) + \frac{1}{\sigma_3^2}(x_3^2 - 2\mu_3 x_3 + \mu_3^2)]\} \\
&= \exp\{-\frac{1}{2}[x_3^2(\frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}) - 2x_3(\frac{k_2-\mu_2}{\sigma_2^2} + \frac{\mu_3}{\sigma_3^2}) + \frac{(k_2-\mu_2)^2}{\sigma_2^2} + \frac{\mu_3^2}{\sigma_3^2}]\} \\
\text{Let } \gamma &= \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2} \text{ then} \\
[x_3|k_2, \mu_2, \mu_3, \sigma_2^2, \sigma_3^2] &= \exp\{-\frac{\gamma}{2}[x_3^2 - \frac{2x_3}{\gamma}(\frac{k_2-\mu_2}{\sigma_2^2} + \frac{\mu_3}{\sigma_3^2}) + \frac{1}{\gamma}(\frac{(k_2-\mu_2)^2}{\sigma_2^2} + \frac{\mu_3^2}{\sigma_3^2})]\} \\
&\propto \exp\{-\frac{\gamma}{2}[(x_3 - \frac{1}{\gamma}(\frac{k_2-\mu_2}{\sigma_2^2} + \frac{\mu_3}{\sigma_3^2}))^2]\} \\
&\sim N(\frac{1}{\gamma}[\frac{k_2-\mu_2}{\sigma_2^2} + \frac{\mu_3}{\sigma_3^2}], \frac{1}{\sqrt{\gamma}}) \\
&= N(\frac{k_2}{2}, \frac{1}{\sqrt{2}}) \text{ when } \mu_2 = \mu_3 = 0 \text{ and } \sigma_2^2 = \sigma_3^2 = 1
\end{aligned} \tag{A.4}
$$

A.3 Simulating $[x_3, x_4|x_1, x_2]$

Consider the conditional distribution of the form

$$
[x_3, x_4|x_1, x_2] \propto \exp\{-\frac{1}{2\sigma_3^2}(x_3 - \mu_3)^2\} \exp\{-\frac{1}{2\sigma_4^2}(x_4 - \mu_4)^2\} \tag{A.5}
$$

with constraint $x_3 = -(x_1 - x_2) + x_4$. If we let $k_3 = x_1 - x_2$, the above constraint becomes a function of a constant, k_3 and x_4 only, $x_3 = x_4 - k_3$. Substituting this into Equation A.5 produces

$$
\begin{aligned}
[x_4|k_3, \mu_3, \mu_4, \sigma_3^2, \sigma_4^2] &\propto \exp\{-\frac{1}{2\sigma_3^2}(x_4 - k_3 - \mu_3)^2\} \exp\{-\frac{1}{2\sigma_4^2}(x_4 - \mu_4)^2\} \\
&= \exp\{-\frac{1}{2}[\frac{1}{\sigma_3^2}(x_4^2 - 2(k_3+\mu_3)x_4 + (k_3+\mu_3)^2) + \frac{1}{\sigma_4^2}(x_4^2 - 2\mu_4 x_4 + \mu_4^2)]\} \\
&= \exp\{-\frac{1}{2}[x_4^2(\frac{1}{\sigma_3^2} + \frac{1}{\sigma_4^2}) - 2x_4(\frac{k_3+\mu_3}{\sigma_3^2} + \frac{\mu_4}{\sigma_4^2}) + \frac{(k_3+\mu_3)^2}{\sigma_3^2} + \frac{\mu_4^2}{\sigma_4^2}]\} \\
\text{Let } \gamma &= \frac{1}{\sigma_3^2} + \frac{1}{\sigma_4^2} \text{ then} \\
[x_4|k_3, \mu_3, \mu_4, \sigma_3^2, \sigma_4^2] &= \exp\{-\frac{\gamma}{2}[x_4^2 - \frac{2x_4}{\gamma}(\frac{k_3+\mu_3}{\sigma_3^2} + \frac{\mu_4}{\sigma_4^2}) + \frac{1}{\gamma}(\frac{(k_3+\mu_3)^2}{\sigma_3^2} + \frac{\mu_4^2}{\sigma_4^2})]\} \\
&\propto \exp\{-\frac{\gamma}{2}[(x_4 - \frac{1}{\gamma}(\frac{k_3+\mu_3}{\sigma_3^2} + \frac{\mu_4}{\sigma_4^2}))^2]\} \\
&\sim N(\frac{1}{\gamma}[\frac{k_3+\mu_3}{\sigma_3^2} + \frac{\mu_4}{\sigma_4^2}], \frac{1}{\sqrt{\gamma}}) \\
&= N(\frac{k_3}{2}, \frac{1}{\sqrt{2}}) \text{ when } \mu_2 = \mu_3 = 0 \text{ and } \sigma_2^2 = \sigma_3^2 = 1
\end{aligned}
$$

(A.6)

Contact Us

Phone: 1300 363 400
+61 3 9545 2176

Email: enquiries@csiro.au

Web: www.csiro.au

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